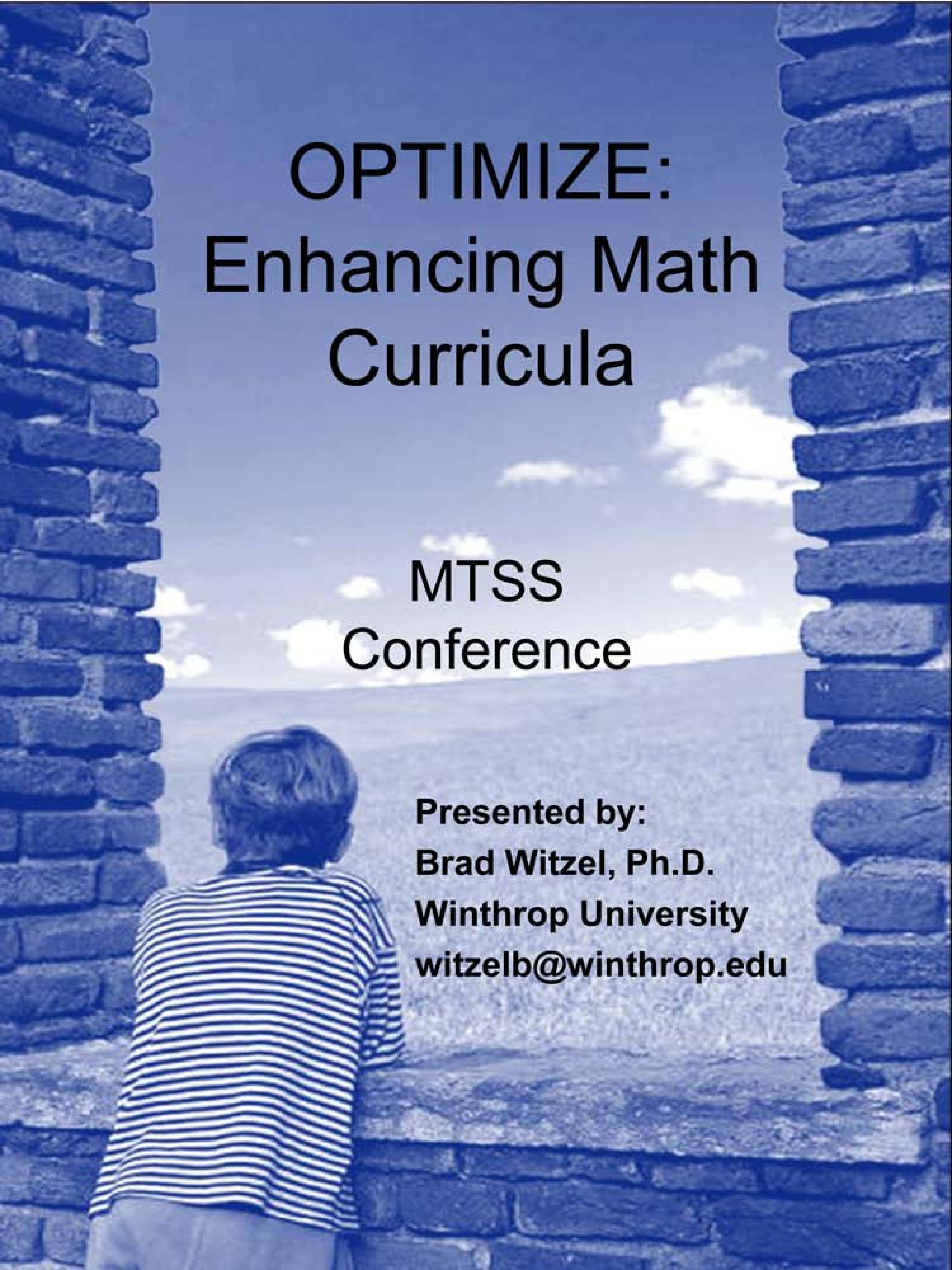
OPTIMIZE:
Enhancing Math
Curricula
MTSS
Conference
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OPTIMIZE: Enhancing Math Curricula

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Mathematics Textbooks

"The single most important factor in predicting whether or not a teacher will be effective is whether the curriculum that is delivered to students in his or her classroom is linked logically or empirically to the outcomes that are desired."

(Berliner, 1986p. 128-129).

Instructional Components of Textbooks

(Taken from Witzel & Chard and Stein et al)

Seven instructional and curriculum components emerged.

A - prioritize essential skills and strategies
B - provide continuous maintenance of previous skills and concepts
C - teach skills in a meaningful context with a focus on conceptual learning
D - sequence skills in a logical manner
E - demonstrate and build instruction using visuals
F - present comprehensive guidance within the teaching manual for providing effective and efficient instruction for students with special needs
G - include a clearly defined intervention program that is both supportive and useful

Mathematics Textbooks

(adapted from Riccomini & Witzel, 2011)

- Math textbooks influence a majority of classroom activities
  - Especially for new teachers and teachers who have not been well prepared to teach mathematics
  - Teachers with poor math content expertise
  - Lack of professional development in the are of mathematics (Malzahn, 2000)
- Textbooks are the primary means of presenting new content

Textbooks Make a Difference

- Curriculum design influences student performance (e.g., Woodward, Baxter, & Robinson, 1999).
  - Students receiving the more explicit approach significantly outperformed the students instructed with *Everyday Mathematics*
- Mathematica Policy Research (Agodini, et al., 2009) analyzed four elementary textbooks. Findings:
  - Achievement was significantly higher in schools assigned to Math Expressions and Saxon Math than in schools assigned to Investigations and Scott Foresman-Addison Wesley Mathematics.
  - Better performing programs led to higher achievement for several student subgroups, including students in schools with low math scores and students in schools with high poverty levels.
- Modifications to mathematics texts are essential for students who are struggling

Selecting Mathematics Textbooks

Needs within Math Textbook Adoption

- ***Match*** the learning needs of students with that of the textbook approach
- Provide intervention and acceleration ***options***
- ***Balance*** conceptual knowledge or procedural knowledge
- Design ***accessibility*** into instruction to meet the needs of low performing students and students with disabilities

Evaluating Your Math Curriculum

- Reviewing and modifying math curricula is ***essential***, but not necessarily ***sufficient*** to produce improved mathematical performance.
- We also need well-managed classrooms and proper implementation (*instructional delivery*) of well-designed programs

Evaluate Your Math Curriculum

a) Program Goals and "Big" Ideas
b) Strand Design vs Spiraled
c) CCSS Math Practices
d) OPTIMIZE
e) Instructional Requirements
f) CRAMATH

Program Goals

Program Goals/Objectives

1. *Are the "Big Ideas" in the program obvious?*
2. *Do the "Big Ideas" match the CCSS?*

A Big Idea is:

- Predictive of concept and skill acquisition and later math achievement (i.e., place value, equivalence)
- Something we can teach and improves student outcomes (i.e. achievement) when we teach it
- A significant amount of instructional time should be devoted to the development, relationships, and practice of these big ideas

Goals and Objectives

Program Goals/Objectives (cont)

Are objectives stated as observable behaviors?

- Objectives help teachers determine student outcomes and should easily connect to district/state standards
- Must be linked to the program assessment procedures.

Do the goals and objectives match the CCSS Math Practices?

Common core "process standards".. Oops, I mean math practices

Practice Standards (NCM called these process standards)

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Program Design

Program Type

1. *Does the program use a strand or spiral design?*
   - Programs using a *strand design* teach fewer topics over a longer period of time while promoting mastery.
   - Programs using *spiral design* present larger number of topics for a short period of time with relatively little depth of integration
2. *Is there a balance between computation instruction and problem solving instruction?*

Balance Computation and Problem Solving

Is there a balance between computation instruction and problem solving instruction?

- The two are not mutually exclusive
- *"…the demand for precision and fluency in the execution of basic skills in school mathematics runs counter to the acquisition of conceptual understanding"* (A Bogus Dichotomy, Wu, 1999 p. 14)

OPTIMIZE – another look at pacing guides

Assumed sequence	Textbook sequence match	Resources	Final sequence
1. Check fluency of basic facts with signed numbers 2. Relevance through actual equations by substitution 3. Equations by subtraction (include actual formulas) 4. Equations by addition (include actual formulas) 5. Equations by division (coefficients) (include actual formulas) 6. Equations by multiplication (include actual formulas) 7. Equations with coefficients and addition or subtraction (include actual formulas) 8. Unknowns on both sides of the equal sign (include actual formulas) 9. Decimal equations 10. Substitution with decimal equations	1. Solving equations using subtraction and addition 2. Solving equations using multiplication and division 3. Solving multi-step equations 4. Solving equations with variables on both sides 5. Multiple calculations of the variable on both sides 6. Solving decimal equations 7. Actual formulas with variables including fractions 8. Ratios and rate equations 9. Percent equations	Previous textbook had a slower pace Multisensory Algebra Guide showed other ways to teach fractions Textbook from Previous Grade included some of the precursor skills her still students lacked Algebraic Thinking for relevance discussion	1. Check fluency of basic facts with signed numbers 2. Relevance through supplementary guide activity 3. Equations by subtraction (include actual formulas) 4. Equations by addition (include actual formulas) 5. Equations by division (coefficients) (include actual formulas) 6. Teach fractions (reducing and multiplying) 7. Equations by multiplication (fractional coefficients) (include actual formulas) 8. Equations with coefficients and addition or subtraction (include actual formulas) 9. Unknowns on both sides of the equal sign (include actual formulas) 10. Decimal equations 11. Substitution with equations

Final sequence

- Begin charting your task analysis and how it matches the textbook.
- Example, reducing expressions

1. Addition and subtraction (preliminary skills)
2. Addition with signed numbers (teacher includes)
3. Distributive property (preliminary skills but with some revisiting)
4. Recognize like variables v. numerals (included)
5. Two variables with one coefficient (teacher includes)
6. Two variables with multiple coefficients and distribution (included)
7. Three variables with multiple coefficients and distribution (included)
8. Over three variables with multiple coefficients and distribution (included)

OPTMIZE

(Riccomini & Witzel, 2011; Witzel & Riccomini, 2005)

O	Order the math skills of a textbook chapter before teaching
P	Pair your sequence with that of the textbook
T	Take note of the similarities and differences
I	Inspect earlier chapters to see if they cover the differences. Check later chapters to see if they cover differences
M	Match supplemental guides to see if they cover the differences
I	Identify additional instruction to complement the current text
Z	Zero in the optimal sequence with your new knowledge
E	Evaluate and improve the sequence every year

ORDER the math skills of a textbook chapter before teaching

Purpose: organize instructional sequence into perceived ideal sequence (most efficient & effective)

Steps

1. Examine first lesson and compare to last lesson
2. List concepts and skills required to achieve objectives

Look for:

- Coherence of critical concepts and component skills
- Include all necessary component skills to achieve objectives

PAIR your Sequence with that of the Textbook

Purpose: Systematic and critical comparison of the two sequences

- Your ORDER and the publisher's sequence

Steps

1. List the sequence in the textbook
2. Side-by-side pairing and review/comparison of the two sequences
3. Identify similarities and differences
4. Identify potential areas requiring adaptations and modifications

Look for:

- Planned overlap of most important skills and concepts
- Sufficient time allocated to instruction and practice
- Large gaps or missing component skills

TAKE note of the Similarities and Differences

Purpose: To identify similarities and differences in the two sequences to help plan and develop modifications and adaptations

Steps

1. Identify similarities and differences
2. Identify potential areas requiring adaptations and modifications

Look for:

- potentially problematic tasks for some students
- Target the concepts and skills that might be most difficult for students and require significant modification

INSPECT and CHECK earlier and later chapters for coverage

Purpose:

1. Inspect earlier chapters to see if they cover the differences.
2. Check later chapters to see if they cover differences

Look for:

- Concepts and skills that are not addressed in earlier chapters
- For example: Previous chapters: 1/4 ÷ 1/2
  Current chapter: 2/97 ÷ 31/17

MATCH supplements and other available curriculum

Purpose: To examine the program's supplemental guides to see if they cover the differences

Steps

1. Following the publisher's directions, find the appropriate supplement and compare concepts and skills covered in supplements (e.g., Extra Practice, Enrichment, Remediation, etc)

Look for:

- Important component skills and relationships between Big Ideas
- Availability of supplements

IDENTIFY additional instructional ideas

Purpose: To identify additional instruction to complement textbook (i.e., DIFFERENTIATE instruction for those students who might struggle)

Steps

1. Following the publisher's directions, find the appropriate supplement and compare concepts and skills covered in supplements (e.g., Extra Practice, Enrichment, Remediation, etc)
2. Design instruction following components of effective instruction

Look for:

- Important component skills and relationships between Big Ideas
- Explicitness of "differentiated" instruction

ZERO in the optimal sequence..

Purpose: To merge best of both sequences to form most efficient and effective teaching sequence.

Steps

1. Relay on your content expertise in math and your experience with learners characteristics.
2. This is not an exact science, so give it a try!!!!
3. Consult relevant scientific research available

Look for:

- Student's types of errors or specific areas of difficulty

Evaluate and Improve often

Purpose: To continuously evaluate and improve sequence every year

Steps

1. Monitor student progress towards unit goals and objectives
2. Evaluate students' achievement on high stakes assessment
3. Review students' performance and analyze for error patterns and deficit areas within and across the Big Ideas
4. Formulate plan to improve sequence for the next time

Look for:

- A better organized instructional sequence that is more beneficial to ALL students

10 instructional principles linked to effective instruction of new material

1. Begin a lesson with a short review of previous learning
2. Present new information in small steps with student practice after each step
3. Ask a large number of questions and check the responses of all students
4. Provide models
5. Guide student practice
6. Check for student understanding
7. Obtain a high success rate
8. Provide scaffolds for difficult tasks
9. Require and monitor independent practice
10. Engage students in weekly and monthly review

(Rosenshine, 2012, p.12)

Selecting Precursor skills

- **In general** - Investigate your state standards to gain an assumption on sequencing and task analysis. Experience with your students provides more information on your needs.
- Teachers are cautious about teaching students when they have not yet learned well known precursor skills.
- However, do not assume grade level lists are the only answer for your students

Scaffolding Instruction

The National Mathematics Advisory Panel stated that "Explicit systematic instruction typically entails teachers explaining and demonstrating specific strategies and allowing students many opportunities to ask and answer questions and to think about the decisions they make while solving problems" (p.48).

Explicit Teaching Steps

Best Practices consistent in math research literature:

- Advance Organizer
- Model
- Guided Practice
- Independent Practice
- Feedback
- Maintenance and Generalization
- Application

Name the components of explicit instruction from:

-Multiplying negatives through Khan
http://www.khanacademy.org/math/arithmetic/negative-numbers/v/why-a-negative-times-a-negative-is-a-positive

-PA DOE on modeling "Teaching Matters"
http://video.search.yahoo.com/search/video;_ylt=A2KLqIDiTzFP4AsARBn7w8QF;_ylu=X3oDMTBncGdyMzQ0BHNlYwNzZWFyY2gEdnRpZAM-?p=explicit+instruction+education&ei=utf-8&n=21&tnr=21

Modeling:

- Teachers' think aloud
  - Linear Algebra - determinants
    http://www.youtube.com/watch?v=36LFsLC3DG8&list=PL9267B3FB749DA276&index=4&feature=plcp
  - Marzock's Left Hands Rule
    http://www.bing.com/videos/search?q=hands+on+trigonometry&view=detail&mid=5F99EA5C353D0874BEF55F99EA5C353D0874BEF5&first=0
  - Calculus - derivatives
    http://www.youtube.com/user/EducatorVids2?v=rqOuTGjp79E

Guided Practice

- Inter-scaffolding from one part of a lesson to another
- Teacher to Student
  - Verbal interactions
  - Notecard call-outs
  - White board
  - Groups and individuals

Let's try some examples

- First some modeling "I do it"
- Then well work together on some "We do it"
- Then you try some on your own "You do it"

Negative Numbers example

- I do it
  - (1/2)÷(3/4) = _____ (2/3) ÷(4/3) = _____
- We do it
  - (1/5)÷(1/4) = _____ (1/3) ÷(2/3) = _____
  - (3/8)÷(1/2) = _____ (1/2) ÷(2/4) = _____
- You do it
  - (1/3)÷(2/5) = _____ (2/3) ÷(1/5) = _____
  - (3/5)÷(1/8) = _____ (3/4) ÷(2/3) = _____

Models of Information Transfer

Cecil Mercer

- "I do it"
- "We do it"
- "You do it"

Matt McGue

The 5 steps to Apprenticeship

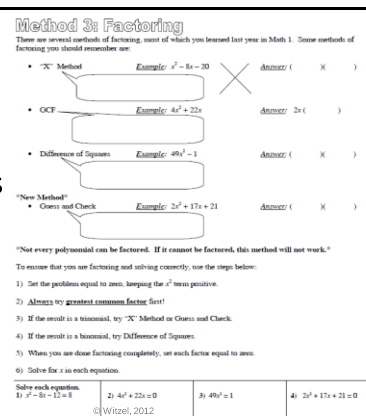
1. "I do, you watch, we talk"
2. "I do, you help, we talk"
3. "You do, I help, we talk"
4. "You do, I watch, we talk"
5. "You do, someone else watches"

2012-2013 Project: Student-created Video Modeling

1. Teach a difficult skill to mastery
2. Present a problem for student to independently solve
3. Video students solving the problem and explaining their reasoning
4. Show the problem solving to others in the same class
5. Use these videos for future classes

What are the potential effects?

Develop stepwise graphic organizers



Method 3: Factoring

There are several methods of factoring, most of which you learned last year in Math 1. Some methods of factoring you should remember are:

- "X" Method — *Example:* $x^2 - 8x - 20$ — *Answer:* ( )( )
- GCF — *Example:* $4x^2 + 22x$ — *Answer:* 2x ( )
- Difference of Squares — *Example:* $49x^2 - 1$ — *Answer:* ( )( )

New Method

- Guess and Check — *Example:* $2x^2 + 17x + 21$ — *Answer:* ( )( )

Not every polynomial can be factored. If it cannot be factored, this method will not work.

To ensure that you are factoring and solving correctly, use the steps below:

1) Set the problem equal to zero, keeping the x^2 term positive.
2) Always try greatest common factor first!
3) If the result is a trinomial, try "X" Method or Guess and Check.
4) If the result is a binomial, try Difference of Squares.
5) When you are done factoring completely, set each factor equal to zero.
6) Solve for x in each equation.

Solve each equation.

1) $x^2 - 8x - 12 = 8$	2) $4x^2 + 22x = 0$	3) $49x^2 = 1$	4) $2x^2 + 17x + 21 = 0$

Miller, teacher,
Oconee County HS
http://moodle.oconee.k12.ga.us/course/view.php?id=810&topic=7

Teach examples from nonexamples

Adapted from CONCEPT DIAGRAM from Bulgren, Shumaker, and Deshler (2003)

Key Terms
- -Numerator
- -Denominator
- -Reduce

Concept Title: fractions

Concept Description: Multiplication of fractions

Examples
- $(1/2)*(3/4)=(1*3)/(2*4)=3/8$
- $(5/4)*(2/3)=(5*2)/(4*3)=10/12$
- $(1/3)*(4/2)=(1*4)/(3*2)=4/6$

Nonexamples DO NOT DO
- $(1/2)*(3/4)= (2/4)*(3/4)=(1+3)/(4)=4/4$
- $(1/2)*(3/4)=(1/2)*(4/3)=(1*4)/(2*3)=3/8$

Always Do: Multiply numerators and denominators separately

Sometimes Do: Reduce fractions to their simplest form

Never Do: Invert and multiply or treat multiplication as addition

Explain the Process: When multiplying fractions complete operations on numerators and denominators separately

Match Interactions to Assessment

- Explicit instruction aligns cleanly with assessment. A 4 step problem counts for 5 points.
- Constructivist instruction leads to applied outcomes. Assess students according to their reasoning and thoughtfulness.

Grading Example

Steps to solving for unknowns

- $4x - 2y = 8$, solve for y — *identify the variables*
- $4x - 2y = 8$ — *add and subtract addends on the variable side*
  -4x -4x
- $\underline{0 - 2y} = \underline{8} \ \underline{-4x}$ — *multiply and divide coefficient*
  -2 -2 -2 -2
- 1y = -4 + 2x — *check reasonableness*

Solving for unknowns

from Riccomini

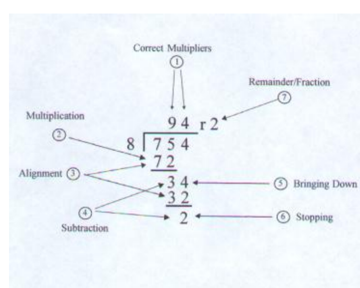
4 Separate Steps with computation check

1. Identify the variables
2. Add and subtract addends on the variable side
3. Multiply and divide coefficients
4. Check reasonable
5. Computation accuracy: Multiplication and division
6. Computation accuracy: Addition and subtraction
7. Computation accuracy: Rational Numbers

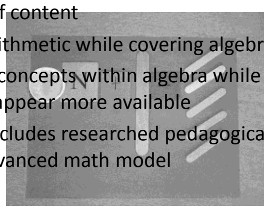
Grading procedures

	ID variables	Add and subtract	Coefficient	Reasonableness	Comput e: +; -	Comput e: multi and div	Comput e: Rational	Answer
Ardell	√	√	√	√	√	√	√	√
Michael	√	√	√	√	√	√	X	X
Brandon	√	√	√	√	√	√	√	√
Manuel	√	√	X	√	√	√	√	X
Miguel	√	√	√	√	√	√	√	√
Said	√	√	√	√	√	√	√	√
Tarek	√	√	√	√	√	√	√	√
Jason	√	√	√	√	√	√	X	X
Revis	√	√	X	√	√	X	√	X

Division Problem



Multisensory Algebra instruction



- CRA sequence allows hands-on and pictorial exploration of content
- Reinforces arithmetic while covering algebra
- Enforces the concepts within algebra while making the solution appear more available
- Instruction includes researched pedagogical steps as well as an advanced math model

What else can we do with CRA?

Measurement (9-12) - uses concrete and graphic models to derive formulas for finding rate, distance, time, angle measures, and arc lengths.

Implement CRA instruction in your classroom. Here's how:

- **C**hoose the math topic to be taught
- **R**eview abstract steps to solve the problem
- **A**djust the steps to eliminate notation or calculation tricks
- **M**atch the abstract steps with an appropriate concrete manipulative
- **A**rrange concrete and representational lessons
- **T**each each concrete, representational, and abstract lesson to student mastery (accuracy without hesitation)
- **H**elp students generalize learning through word problems and problem solving events

Choose a math topic

- Plan what is to be taught ahead of time.
- Group lessons according to the big idea as determined by your state standards
- Sequence the lessons so they start basic and gradually introduce new topics
- Any math topic can be examined for CRA linkage

Review the abstract steps used to solve the problem

- What is the desired math outcome of the group of lessons?
- Determine the procedural goal of the combination of math skills
- List out the steps or procedures
- Remember, not all math skills require abstract knowledge.

Adjust the steps to eliminate notation or calculation tricks

- Change or modify steps to create the most logical and sequential set of procedures
- Take a child's point of view when reviewing steps

Match the abstract steps with an appropriate concrete manipulative

- Initial understanding of content will be based on interactions with concrete objects, so be careful which ones you choose.
- The conceptual effectiveness of the manipulative object should be noted in accordance to the math skill being taught.
- Avoid concrete objects that only cover a few skills.
- You may have to teach two stages of concrete knowledge
- Also, not all concrete objects are appropriate for CRA instruction. Some materials are effective for conceptual growth while others are useful for procedural work.

Arrange concrete and representational lessons

- Practice concrete manipulations. The same questions that you encounter you can be certain your students will as well.
- Practice how to mark pictorial representations that appear similar to concrete manipulations.
- Make certain that your language throughout instruction matches the language required for the desired outcome.

Teach each CRA lesson to mastery

- Model and guide students in their use of manipulative objects and pictorial representations.
- Teach students step by step gradually introducing mathematical vocabulary. Allow students to name or invent their stepwise procedures within instruction.
- Move from concrete to representational to abstract learning levels only after students show accuracy without hesitations in manipulations or drawings.
- Assess each level of learning according to stepwise procedures. Take account of students who created different procedures.

Help students generalize what they learn through word problems

- Incorporate word problems throughout a lesson to help show social relevance as to why a math skill is important to learn
- Use language experiences through the learning process to help prepare for word problem and problem solving application

Conclusion

- What are some things to consider when reviewing textbooks?
- What is OPTIMIZE?
- What is CRAMATH?

Good luck with your curricular adaptations to CCSS!

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