MS and HS Algebra for Students Struggling in Math

MTSS Conference

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Middle and High School Algebra for Students Struggling in Math

MTSS
½ day

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In a game, exactly six inverted cups stand side by side in a straight line, and each has exactly one ball hidden under it. The cups are numbered consecutively 1 through 6. Each of the balls is painted a single solid color. The colors of the balls are green, magenta, orange, purpose, red and yellow. The balls have been hidden under the cups in a manner that conforms to the following conditions:
The purple ball must be hidden under a lower-numbered cup than the orange ball.
The red ball must be hidden under a cup immediately adjacent to the cup under which the magenta ball is hidden.
The green ball must be hidden under cup 5.

1. Which of the following could be the colors of the balls under the cups, in order from 1 through 6?
(A) Green, yellow, magenta, red, purple, orange
(B) Magenta, green, purple, red, orange ,yellow
(C) Magenta, red, purple, yellow, green, orange
(D) Orange, yellow, red, magenta, green, purple
(E) Red, purple, magenta, yellow, green, orange

2. If the magenta ball is under cup 4, the red ball must be under cup
(A) 1 (B) 2 (C) 3 (D) 5 (E) 6

Arithmetic to Algebra Gap
(Witzel, Smith, & Brownell, 2001)

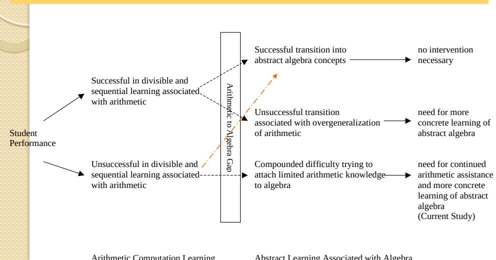

Figure 1. Flowchart of Algebraic Needs for Students Who Experience Difficulty in Math

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Nationally, what do algebra teachers say? (NMP, 2008)

743 algebra teachers in 310 schools nationally responded to a survey on algebra instruction and student learning in 2007.

Findings:

- The teachers generally rated their students' background preparation for Algebra I as weak. The three skill areas in which teachers reported their students have the poorest preparation are rational numbers, word problems, and study habits
- Regarding the best means of preparing students, 578 suggested a greater focus on mastery of elementary mathematical concepts and skills
- Teachers were less excited about how current textbook approaches meet the needs of diverse student populations

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More findings from the NSAT

- Use of calculators was quite mixed with 33% saying they never use them and 31% use them frequently (more than once a week)
- 60% use physical tools less than once a week and only 9% use them frequently
- 51% consider "mixed-ability" grouping to be a moderate or serious problem with instruction
- The greatest challenge to teachers was #1 – "working with unmotivated students." This was chosen by 58% of the middle school teachers and 65% of the high school teachers. The next most frequent response was "making mathematics accessible and comprehensible to all my students," selected by 14% of the middle school teachers and 9% of the high school teachers.

For Today

- Reviewing SBAC Algebra questions
- Effort and achievement
- Build from arithmetic and teach the details
- Instructional Review
  - Explicit Instruction
  - CRA
- Provide ample opportunities to learn

8th grade – radicals and integers

The average distance from Jupiter to the Sun is about 5×10^8 miles. The average distance from Venus to the Sun is about 7×10^7 miles.

The average distance from Jupiter to the Sun is about how many times as great as the average distance from Venus to the Sun?

[ ] times

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8th grade –radicals and integers

Select **all** of the expressions that have a value between 0 and 1.

(A) $8^7 \cdot 8^{-12}$

(B) $\frac{7^4}{7^{-3}}$

(C) $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^9$

(D) $\frac{(-5)^6}{(-5)^{10}}$

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8th grade - equations

Three students solved the equation $3(5x-14)=18$ in different ways, but each student arrived at the correct answer. Select **all** of the solutions that show a correct method for solving the equation.

(A)
$3(5x-14)=18$
$8x-14 = 18$
$+14 \quad +14$
$\frac{8x}{8}=\frac{32}{8}$
$x=4$

(B)
$\frac{1}{\cancel{3}}\cdot\cancel{3}(5x-14)=18\cdot\frac{1}{3}$
$5x-14 = 6$
$+14 \quad +14$
$\frac{5x}{5}=\frac{20}{5}$
$x=4$

(C)
$3(5x-14)=18$
$\frac{15x}{15}-\frac{42}{15}=\frac{18}{15}$
$+\frac{42}{15} \quad +\frac{42}{15}$
$x=\frac{60}{15}$
$x=4$

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8th grade – equations and expressions

For each linear equation in this table, indicate whether the equation has no solution, one solution, or infinitely many solutions.

Equation	No Solution	One Solution	Infinitely Many Solutions
$7x + 21 = 21$			
$12x + 15 = 12x - 15$			
$-5x - 25 = 5x + 25$			

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8th grade – radicals

Classify the numbers in the box as perfect squares and perfect cubes. To classify a number, drag it to the appropriate column in the chart. Numbers that are neither perfect squares nor perfect cubes should **not** be placed in the chart.

1 64 96 125 200 256 333 361

Perfect Squares but Not Perfect Cubes	Both Perfect Squares and Perfect Cubes	Perfect Cubes but Not Perfect Squares

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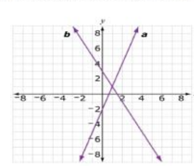
8th grade - graphing

The graphs of line *a* and line *b* are shown on this coordinate grid.



Match each line with its equation. Click on an equation and then drag it to the corresponding box for each line.

The equation of line *a* is [ ]

The equation of line *b* is [ ]

$y = -2x + 3$ $\quad y = 2x + 3$ $\quad y = 3x - 2$

$y = -\frac{1}{2}x + 3$ $\quad y = -\frac{1}{3}x - 2$

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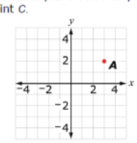
8th grade - graphing

Point A is plotted on the xy-coordinate plane below. You must determine the location of point C given the following criteria:

- Point C has integer coordinates.
- The graph of line $\overline{AC}$ is **not** a function.

Click on the xy-coordinate plane below to place a point that could represent point C.



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8th grade – functions (definitions)

Fill in each x-value and y-value in the table below to create a relation that is **not** a function.

x	y

Sample Top-score Response:

x	y
4	0
4	1
4	2
4	3
4	4

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Scoring Rubric:

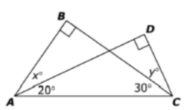
Responses to this item will receive 0-1 point, based on the following:

1 point: The student shows a thorough understanding of the definition of a function. The student creates a relation that is not a function.

0 points: The student shows no understanding of the definition of a function. The student creates a relation that is a function.

8th grade - geometry

Right triangle ABC and right triangle ACD overlap as shown below. Angle DAC measures 20º and angle BCA measures 30º.



not drawn to scale

What are the values of x and y?

x = ☐ degrees y = ☐ degrees

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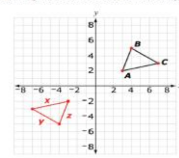
8th grade – similarity and congruence

Triangle ABC on this coordinate grid was created by joining points A (3, 2), B (4, 5), and C (7, 3) with line segments.

Triangle ABC was reflected over the x-axis and then reflected over the y-axis to form the red triangle, where x, y, and z represent the lengths of the sides of the red triangle.



Click the appropriate boxes in the table to show which sides of the triangles have equal lengths.

	x	y	z
AB			
AC			
BC			

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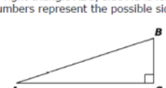
8th grade – Pythagorean theorem

In right triangle ABC, side AC is longer than side BC. The boxed numbers represent the possible side lengths of triangle ABC.



7	8
15	17
18	20
24	25

not drawn to scale

Identify three boxed numbers that could be the side lengths of triangle ABC. Enter the number you chose to represent the length of each side.

1a. BC = ☐

1b. AC = ☐

1c. AB = ☐

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8th grade – Pythagorean theorem

What is the distance between (0, 0) and (8, 15) on the xy-coordinate plane?

Ⓐ 7 units

Ⓑ 8 units

Ⓒ 17 units

Ⓓ 23 units

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HS – computation

For items 1a – 1e, determine whether each equation is True or False.

1a. $\sqrt{32} = 2^{\frac{5}{2}}$ Ⓣ True Ⓕ False

1b. $16^{\frac{3}{2}} = 8^2$ Ⓣ True Ⓕ False

1c. $4^{\frac{1}{2}} = \sqrt[4]{64}$ Ⓣ True Ⓕ False

1d. $2^8 = \left(\sqrt[3]{16}\right)^6$ Ⓣ True Ⓕ False

1e. $\left(\sqrt{64}\right)^{\frac{1}{3}} = 8^{\frac{1}{6}}$ Ⓣ True Ⓕ False

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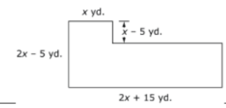
Algebra – Q1, part 1

Part A

A town council plans to build a public parking lot. The outline below represents the proposed shape of the parking lot.



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Algebra – Q1, part 2

Write an expression for the area, in square feet, of this proposed parking lot. Explain the reasoning you used to find the expression.

Part B

The town council has plans to double the area of the parking lot in a few years. They create two plans to do this. The first plan increases the length of the base of the parking lot by *p* yards, as shown in the diagram below.

Write an expression in terms of *x* to represent the value of *p*, in feet. Explain the reasoning you used to find the value of *p*.

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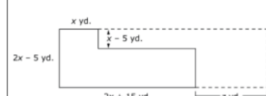
Algebra – Q1, part 3

Part C

The town council's second plan to double the area changes the shape of the parking lot to a rectangle, as shown in the diagram below.



Can the value of *z* be represented as a polynomial with integer coefficients? Justify your reasoning.

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Algebra – Q2, part 1

Your Assignment:
In this task you will assume the role of consultant for a farmer. You will analyze the options available to the farmer for handling the storage of shelled field corn (shown in the pictures above). In the past, the farmer has sold the corn as it was harvested, and did not store the corn to be sold in the future. The farmer has increased the number of acres used to grow corn, and now is exploring the cost of storing the corn until after the harvest is complete and then selling it. You will analyze two storage options available to the farmer for storing the grain that is harvested.

- The corn can be stored in grain bins constructed on the farm.
- The corn can be stored in rental storage close to the farm.

Following the tasks, you will recommend which type of storage the farmer should use.

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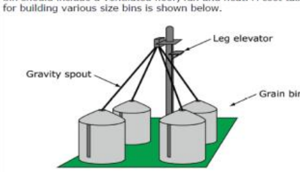
Algebra – Q2, part 2

Corn Storage

Session 1

Part A

Your first job is to determine the most efficient cost for constructing 4 grain bins which can be used to store the harvested corn. A leg elevator, which moves the corn from ground level into the bins, must also be built. The bins must be able to hold the 132,000 bushels of corn from the harvest. Each bin should include a ventilated floor, fan and heat. A cost table for building various size bins is shown below.



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Algebra-Q2, part3

Cost of Grain Bins

Diameter (feet)	Height (feet)	Capacity (bushels)	Cost Without Floor ($)	Add for: Concrete Floor ($)	Add for: Steel Floor ($)	Add for: Ventilated Floor ($)	Add for: Fan and Heat ($)
30	19	10,775	11,525	1,650	1,130	4,250	2,950
	24	13,625	16,000	1,775	1,130	4,250	2,950
	32	18,175	20,100	1,975	1,130	4,250	2,950
33	24	16,475	17,725	2,050	1,320	5,100	3,025
	27	18,550	20,050	2,100	1,320	5,100	3,025
	32	21,975	24,950	2,550	1,320	5,100	3,025
36	24	19,625	21,575	2,575	1,540	6,000	3,075
	27	22,075	23,475	2,675	1,540	6,000	3,075
	32	26,150	26,100	2,775	1,540	6,000	3,075
	40	32,700	28,925	2,950	1,540	6,000	3,075
42	27	30,050	28,450	3,650	2,065	8,100	3,225
	32	35,600	32,525	3,875	2,065	8,100	3,225
	40	44,500	39,650	4,075	2,065	8,100	3,225
	48	53,425	47,200	4,400	2,065	8,100	3,225
48	27	39,250	41,150	4,775	2,640	10,450	3,350
	32	46,500	48,900	5,050	2,640	10,450	3,350
	40	58,150	55,000	5,300	2,640	10,450	3,350
	48	69,775	61,650	5,750	2,640	10,450	3,350

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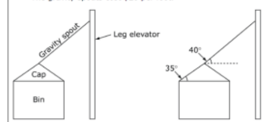
Algebra, Q2, part4

All 4 bins must have the same capacity.

The bins must be built to the following specifications.

- The height listed in the table does not include the height of the conical cap on top of the bin. The cap forms a 35° angle with the base.
- The distance from the outer edge of the bins to the leg elevator will be 15 feet.
- A gravity spout is placed so that it runs from the top of the cap to a point that is 2 feet below the top of the elevator leg. To account for certain moisture content the gravity spouts will slope 40 degrees to the horizontal.
- The average cost involved in the construction of the leg elevator is $15,000 plus $125 for every foot in height.
- The gravity spouts cost $20 per foot.



Find the most efficient cost of the construction. Be sure to include the bins (caps are included in the price), gravity spouts, and leg elevator.

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Algebra, Q2, part 5

Part B

When corn is harvested, the moisture content varies, but is typically above the level desired for selling or storing corn, so the corn must be dried. The moisture content is given as a percent that represents the proportion of the weight of the corn that is from water. For example, if you had 40 lbs of corn at 25% moisture content, it would consist of 10 lbs of water and 30 lbs of dry material. As corn dries, the amount of water decreases, but the amount of dry material stays the same, so the percent of weight from water will decrease.

The final moisture contents for various purposes are as follows:

- 15.5% to sell at market
- 14.0% to store at a rental storage facility
- 13.5% to store in grain bins constructed on the farm

There is a cost for drying corn to 15.5% moisture to be able to sell it at market, but there is extra cost to dry it below 15.5%. This extra cost is a cost of storage since it must be paid only if the grain is to be stored and not sold at market.

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Algebra, Q2, part 6

Assuming corn is harvested at an initial moisture content of 20% and you use LP gas as fuel for your dryer, use the information in tables 1 and 2 below to calculate the extra cost per bushel of drying corn to a final moisture content of 14% and 13.5%. Justify your answer mathematically and show all the steps in your calculation. You can use the regression tool in the spreadsheet provided if necessary. The BTUs required to dry corn to a final moisture content of 15.5% and 13.5% are not in the table but can be found using the provided regression tool.

Energy (BTU's) Required to Dry a Bushel of Wet Corn

Final Moisture Content	Initial Moisture Content: 20%	22%	24%	26%	28%	30%
17%	5,625	8,744	11,714	14,487	17,086	19,545
16%	7,522	10,596	13,506	16,241	18,784	21,188
15%	9,579	12,589	15,447	18,118	20,624	22,978
14%	11,635	14,582	17,388	19,994	22,463	24,768
13%	13,878	16,774	19,528	22,088	24,486	26,744

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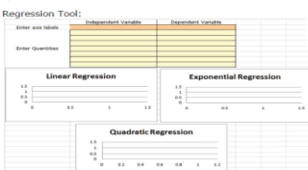
Algebra, Q2, part 7

Energy Content (BTU's) per Unit of Fuel

Fuel Type	Unit	BTU's per Unit of Fuel
Oil	Gallon	140,000
LP gas	Gallon	92,000
Electricity	kWh	3,414
Natural gas	Cubic foot	1,000

To use the regression tool below, enter labels for the axes and pairs of independent and dependent variable values in the spreadsheet.

Regression Tool:



Enter your final answers:
Extra cost to dry 1 bushel of corn to 14% = ______
Extra cost to dry 1 bushel of corn to 13.5% = ______
(Record these values on your note sheet; you will need them in a later part.)

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What is different about these questions from previous assessments?

Student Attitude Matters

- Develop an Internal Locus of Control

Effort over Ability (Woodward, 2011)

- Students who are motivated to work at math have greater task-persistence and long-tem achievement
- Low achieving students have evidence that they don't succeed. Thus they are motivated to show little effort to achieve.
- Use contingent praise based on student effort and graph results of effort to show the effects of effort on achievement. Verbally praise those who work to be engaged.

Engagement: Involve the students as often as possible

- Engagement is essential but often difficult
- Students who struggle early learn to be passive or use avoidance behaviors in math class

1. Create a safe class zone (Allow students multiple ways to ask and answer questions)
2. Make math relevant (Socially and Academically)
3. Instruct in an interactive and interesting manner

Instruction Matters (NMP, 2008)

Research on students who are low achievers, have difficulties in mathematics, or have learning disabilities related to mathematics tells us that the effective practice includes:

- Explicit methods of instruction available on a regular basis
- Clear problem solving models
- Carefully orchestrated examples/ sequences of examples.
- Concrete objects to understand abstract representations and notation.
- Participatory thinking aloud by students and teachers.

Guide student learning

- "Decades of research clearly demonstrate that for novices (comprising virtually all students), direct, explicit instruction is more effective and more efficient than partial guidance" (Clark, Kirschner, & Sweller, 2012, p. 6).
- "..teachers are more effective when they provide explicit guidance accompanied by practice and feedback, not when they require students to discover many aspects of what they must learn" (Clark, Kirschner, & Sweller, 2012, p. 6).

Name the components of explicit instruction from:

-Multiplying negatives through Khan
http://www.khanacademy.org/math/arithmetic/negative-numbers/v/why-a-negative-times-a-negative-is-a-positive

-PA DOE on modeling "Teaching Matters"
http://video.search.yahoo.com/search/video;_ylt=A2KLqIDiTzFP4AsARBn7w8QF;_ylu=X3oDMTBncGdyMzQ0BHNlYwNzZWFyY2gEdnRpZAM-?p=explicit+instruction+education&ei=utf-8&n=21&tnr=21

Modeling:

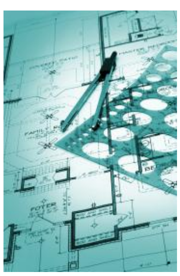
- Teachers' think aloud
  - Linear Algebra - determinants
    http://www.youtube.com/watch?v=36LFsLC3DG8&list=PL9267B3FB749DA276&index=4&feature=plcp
  - Marzock's Left Hands Rule
    http://www.bing.com/videos/search?q=hands+on+trigonometry&view=detail&mid=5F99EA5C353D0874BEF55F99EA5C353D0874BEF5&first=0
  - Calculus - derivatives
    http://www.youtube.com/user/EducatorVids2?v=rqOuTGjp79E

2012-2013 Project: Student-created Video Modeling

1. Teach a difficult skill to mastery
2. Present a problem for student to independently solve
3. Video students solving the problem and explaining their reasoning
4. Show the problem solving to others in the same class
5. Use these videos for future classes

What are the potential effects?

Build from Arithmetic and Teach the Details



Concepts vs Procedures: A Bogus Dichotomy (Wu, 1999)

- Reasoning and Algorithms
- In fact, algorithms should show reasoning.
- When math is a series of tricks, then the argument of concepts vs procedures is valid. Without tricks, then math is logical in reason and procedure.
- Be careful what is called a "trick." Many "tricks" are developed from accurate procedures. It is just a few steps are ignored.

Clues and Undos procedures

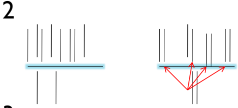
$^5/_6X + 4 = 8$,
solve for X

Clues	Undos
Multiply by $^5/_6$	Divide by $^5/_6$
Add 4	Subtract 4

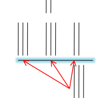
Algebraic - The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions. (MA.D.1.3)
Operations - selects the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers, ratios, proportions, and percents, including the appropriate application of the algebraic order of operations. (MA.A.3.3)

Connect Algebra to Arithmetic: Build on what they know

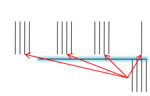
- 8 ÷ 2



- 8 ÷ 3



- 13 ÷ 4



Progressions: Make Sense of the Steps

$4\,{}^{2}/_{5}$ This mixed fraction as an improper fraction is ${}^{22}/_{5}$.

Why?

Tricks are not helpful to students with memory problems, think about the concept and purpose to the calculations

Say, "Four and two – fifths" "And says to add"

${}^{4}/_{1} + {}^{2}/_{5}$ or ${}^{20}/_{5} + {}^{2}/_{5} = {}^{22}/_{5}$.

This is why you multiply the fraction's denominator and then add the numerator.

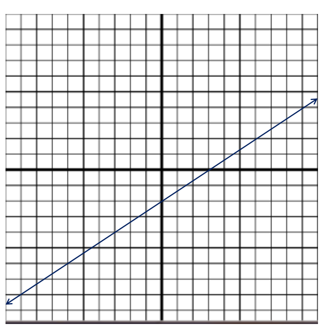
Progressions Division of Fractions

- Why is it that when you divide fractions, the answer is larger? Also, why do you invert and multiply?
- ${}^{2}/_{3}$ divided by ${}^{1}/_{4} = {}^{2}/_{3}\,({}^{4}/_{1}) = {}^{8}/_{3}$

$$\frac{{}^{2}/_{3}\,({}^{4}/_{1})}{{}^{1}/_{4}\,({}^{4}/_{1})} = \frac{{}^{8}/_{3}}{{}^{4}/_{4}} = \frac{{}^{8}/_{3}}{{}^{1}/_{1}} = {}^{8}/_{3}$$

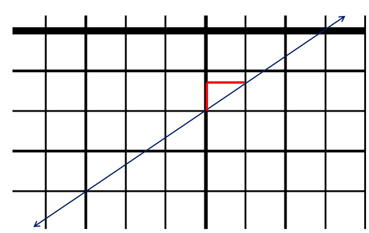
Simple graphing: Y = ${}^{2}/_{3}$x - 2

1) Plot the y-intercept
2) Use the coefficient of x as the slope and go rise (numerator) then run (denominator)
3) Make a point
4) Connect and continue to make a line
5) But wait!



Take a closer look: Y = ${}^{2}/_{3}$x - 2

The rise over run is actually ${}^{2}/_{3}$ over 1



Progressions: Algebra evolves from the basics

Adding with unlike denominators

$$\frac{5 + \frac{1}{y}}{3 + \frac{2}{y^2}} = \frac{\left(\frac{5y}{y} + \frac{1}{y}\right)}{\left(\frac{3y^2}{y^2} + \frac{2}{y^2}\right)} = \frac{\left(\frac{5y+1}{y}\right)}{\left(\frac{3y^2+2}{y^2}\right)}$$

Division of fractions

$$= \frac{\left(\frac{5y+1}{y}\right)\left(\frac{y^2}{3y^2+2}\right)}{\left(\frac{3y^2+2}{y^2}\right)\left(\frac{y^2}{3y^2+2}\right)} = \frac{\frac{y(5y+1)}{(3y^2+2)}}{1}$$

$$= \frac{5y^2+y}{3y^2+2}$$

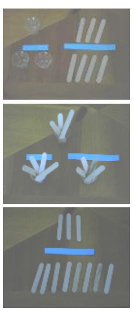
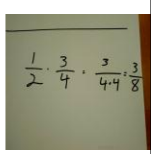
Algebra and CCSS – just a reminder

8th Grade	Algebra
The Number System • Know that there are numbers that are not rational, and approximate them by rational numbers. **Expressions and Equations** • Work with radicals and integer exponents. • Understand the connections between proportional relationships, lines, and linear equations. • Analyze and solve linear equations and pairs of simultaneous linear equations. **Functions** • Define, evaluate, and compare functions. • Use functions to model relationships between quantities. **Geometry** • Understand congruence and similarity using physical models, transparencies, or geometry software. • Understand and apply the Pythagorean Theorem. • Solve real-world and mathematical problems involving volume of cylinders, cones and spheres. **Statistics and Probability** • Investigate patterns of association in bivariate data.	**Seeing Structure in Expressions** • Interpret the structure of expressions • Write expressions in equivalent forms to solve problems **Arithmetic with Polynomials and Rational Expressions** • Perform arithmetic operations on polynomials • Understand the relationship between zeros and factors of polynomials • Use polynomial identities to solve problems • Rewrite rational expressions **Creating Equations** • Create equations that describe numbers or relationships **Reasoning with Equations and Inequalities** • Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Solve systems of equations • Represent and solve equations and inequalities graphically

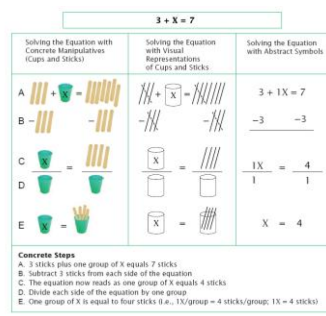
CRA sequence of instruction

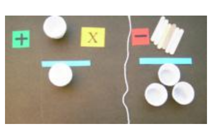
CRA approach

- CRA is the Concrete to Representational to Abstract sequence of instruction.
- Three stages of learning
- C = Learning through concrete hands-on manipulative objects
- R = Learning through pictorial forms of the math skill
- A = Learning through work with abstract (Arabic) notation
- www.rtitlc.org

Concrete	Representational	Abstract
		

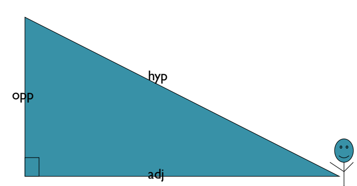
CRA (Gersten et al, p. 35)





Trigonometric ratios

(Willie Ware and Dee Miller)



Why would CRA be effective?

(Witzel, Riccomini, & Schneider, 2008)

- Multimodal forms of math acquisition to aid memory and retrieval
- Multiple learning styles are being met to aid relevance and motivation
- Meaningful manipulations of materials allow students to rationalize abstract mathematics
- Procedural accuracy; provides an alternative to algorithm memorization of math rules
- Transportable without concrete materials

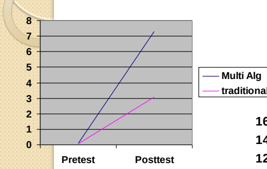
Teach each CRA lesson to mastery

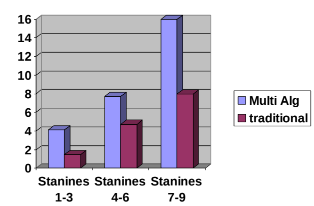
- Model and guide students in their use of manipulative objects and pictorial representations.
- Teach students step by step gradually introducing mathematical vocabulary. Allow students to name or invent their stepwise procedures within instruction.
- Move from concrete to representational to abstract learning levels only after students show accuracy without hesitations in manipulations or drawings.
- Assess each level of learning according to stepwise procedures. Take account of students who created different procedures.

Research Support

- From research studies
- To statewide initiatives
- To individual classrooms

Multisensory Algebra success





Research Support

- Statistic
  - Students with learning difficulties using this model outperformed peers on posttest and follow-up measures ($F=13.89, p<0.000$) (Witzel, Mercer, & Miller, 2003)
  - Students with a history of high math achievement scores also show benefit on the posttest ($F=10.37, p<0.01$) and the follow-up ($F=6.97, p<0.01$) despite pretest favoring of traditional ($F=12.18, p<0.001$) (Witzel, 2005).
- Testimonial
  - Teachers wanted to stop using their current instructional series and textbooks
  - One teacher claimed that he would never teach algebra using any other method than through this model

More studies are ongoing

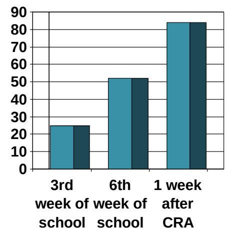
FL scores from the Algebra Success Keys Project

(headed by Dr. Mary Little, PhD, UCF)

- In a FL DOE 2006 call of the critical need to improve student rates of learning where only 4 of 67 districts met AYP in math for students with disabilities, CRA gained much attention for it's potential.
- 5 districts created assessments based on their states benchmarks that mirrored their statewide exams in order to test this model with their teachers
- The results…

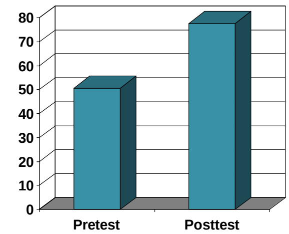
One district math supervisor

He stated that FCAT scores increased and number sense jumped levels to proficient after one year. This was new for this population.

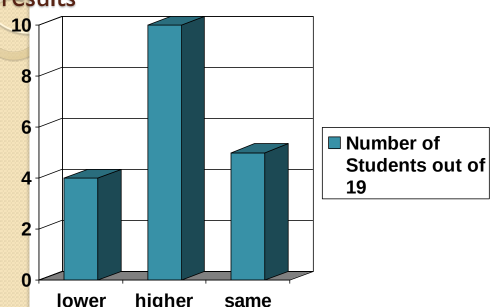


Another math director

This director taught a first year teacher who said she was struggling reaching her children how to implement CRA in her Geometry classes



A third director's pilot looks at FCAT results



Make a change based on results

- Joe McNaughton, math director and National Board Certified teacher, challenged 29 secondary math teachers to make a change with CRA implementation.
- Of 29 teachers to attend, 29 accepted.
- Leading the way were 4 AP Calculus teachers

Choosing Manipulatives and Preparing for Instruction

- Review the abstract problem solving steps and processes before choosing manipulative objects
- Ask yourself:

1. Are these manipulatives easy to use?
2. How can these manipulatives be used for concept and process?
3. Can I follow the same abstract steps using these manipulatives?
4. How will the pictorial representation stage appear with these manipulatives?
5. How many similar math skills can be taught using these manipulatives?

Next Steps with Visual Representations

- Working with manipulatives

  Manipulative objects do not teach children, teachers do. The manipulatives are mere tools to reach an outcome, usually an abstract one.
- Organize the sequence of the instructional steps
- Practice math dialogue to match instructional procedures.

Math Interactions

- **Use language to take students from one level of learning to the next.**
- Ways to increase interactions:
  - Allow students to interact frequently with the class materials and concepts
  - Model and encourage level appropriate math vocabulary in class dialogue
  - Use white boards to assess **step by step** process of concept
  - Set up cooperative groups with systematic interaction
  - Use journals to practice student think alouds and conceptualization

Don't' Give Up! Provide Ample Opportunities to Learn

Next Steps

- Interleaving different types of problems in occasional series of independent practices is superior to only presenting solitary types of problems (Rohrer and Paschel, 2010)
- Provide plenty of instructional modeling, including video models and worked samples within homework.
  - http://www.khanacademy.org/
- Commit to having students learn

Double Dosing (Nomi & Allensworth, 2011)

- In Chicago, each year about ¼ of students fail their ninth grade algebra course. With 9th grade failure associated with dropout, the district tried something
- They double-blocked algebra to give students 2x the amount of instruction time.
- Most students below the 50th %ile enrolled

Double-dosing outcomes

- Increased class interactivity and engaging pedagogy

Improved test scores but not failure rate

- Students who entered with lower test scores scored increased gain scores more than those who entered the treatment with slightly higher scores.
- Failure rates remained the same

"Double-dose algebra was least effective for students entering high school with the weakest math abilities" (Nomi & Allensworth, 2011, p. 181)

System Overload

- Much resistance may occur with teachers.
- Once a teacher team commits to the change, infuse research-supported instruction one step at a time.

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