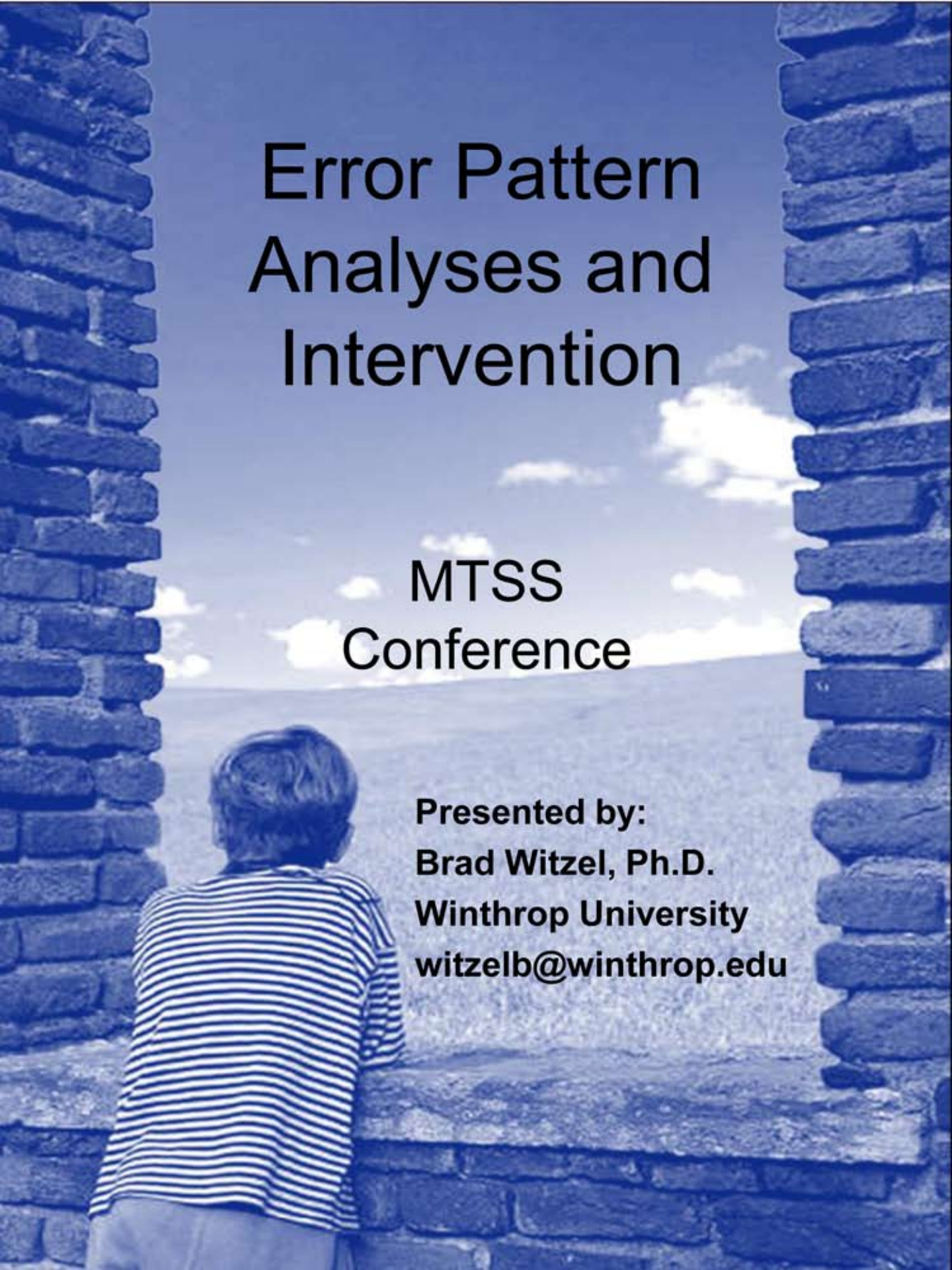

Error Pattern Analyses and Intervention

MTSS Conference

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Error Pattern Analysis and Intervention

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Overview

- Overview of Error Patterns
- Error Analysis
  - Procedures
- Specific Error Patterns
  - Case Study-Error Patterns
    - Fractions
    - Algebra
- Algebra and Beyond
  - Next Steps

Error Pattern Analyses

- As math problems become more complex, students need to go through a series of steps to solve problems.
- Often an error in any of these steps can cause failure in the final response.
- As a result, it is our responsibility to determine what the error is and whether the error is conceptual, procedural, or memory related.

Error Pattern Analysis

- Knowledge of common math errors and misconceptions provides improved opportunities for lesson planning
- Knowing why a student is completing work incorrectly allows for more efficient and effective intervention

Grading Example
Steps to solving for unknowns

- $4x - 2y = 8$, solve for y — *identify the variables*
- $4x - 2y = 8$ (-4x, -4x) — *add and subtract addends on the variable side*
- $\frac{0 - 2y}{-2} = \frac{8 - 4x}{-2}$ (-2, -2) — *multiply and divide coefficient*
- $1y = -4 + 2x$ — *check reasonableness*

Solving for unknowns

4 Separate Steps with computation check

1. Identify the variables
2. Add and subtract addends on the variable side
3. Multiply and divide coefficients
4. Check reasonable
5. Computation accuracy: Multiplication and division
6. Computation accuracy: Addition and subtraction
7. Computation accuracy: Rational Numbers

Grading procedures

	ID variables	Add and subtract	Coefficient	Reasonableness	Compute: +; -	Compute: multi and div	Compute: Rational	Answer
Ardell	√	√	√	√	√	√	√	√
Michael	√	√	√	√	√	√	X	X
Brandon	√	√	√	√	√	√	√	√
Manuel	√	√	X	√	√	√	√	X
Miguel	√	√	√	√	√	√	√	√
Said	√	√	√	√	√	√	√	√
Tarek	√	√	√	√	√	√	√	√
Jason	√	√	√	√	√	√	X	X
Revis	√	√	X	√	√	X	√	X

Steps to Error Pattern Analysis

(adapted from Howell, Fox, Morehead, 1993; KU)

1. Collect at least 3 – 5 samples for each type
2. Have student think aloud during the problem solving process
3. Set up stepwise expectations
4. Analyze student responses for error patterns
5. Identify what is correct vs incorrect
6. Confirm the error pattern with the student
7. Set-up a error-specific intervention

Types of Test-Taking Errors

(Nolting, 1997)

1. Misread directions
2. Careless errors
3. Concept errors
4. Application errors
5. Procedural errors
   - Sections of a test are missed more than others
   - Skipping the last step in problems
   - Second-guessing answers
   - Pacing (rushing or slowing on a problem)
   - Blank answers

- Study errors

Common Math Error Types

Conceptual

Factual

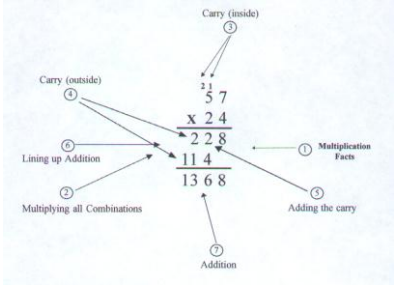
Procedural

Careless

Oftentimes, these error types affect one another

Grading Example

From Riccomini



Multiplication Problem procedures

from Riccomini

7 Separate Steps:

1. **Multiplication Facts:** Are the one-digit multiplication processes completed accurately?
2. **Multiplying All Combinations:** Are all different kinds of multiplication attempted?
3. **Carry (Inside):** Are carries assigned to the proper column?
4. **Carry (Outside):** Is the last carry part of the product?
5. **Adding the Carry:** Are carried numbers combined with the proper column?
6. **Lining up Addition:** Are the intermediate products lined up correctly?
7. **Addition:** Is the final addition process carried out properly.

Grading procedures

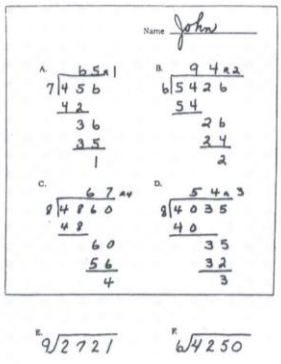
	Multi fact	combo	Carry in	Carry out	Add carry	Line up add	Add facts	Answer
Mike	√	√	√	√	√	√	√	√
Tarek	√	X	√	√	√	√	√	X
Miguel	√	√	√	√	√	√	√	√
Manuel	√	√	X	X	√	√	√	X
Jose	√	√	√	√	√	√	√	√
Pam	√	√	√	√	√	X	√	X
Michele	√	√	√	√	√	√	√	√
Brandon	√	√	√	√	√	√	√	√
Stan	√	√	√	√	√	X	√	X

Breakout Activity

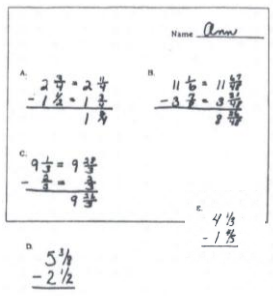
Identify the errors for the grade level of one of the below students that you work with:

1. Elementary Grades:
   John
2. Middle School Grades
   Ann
3. High School Grades
   Juan

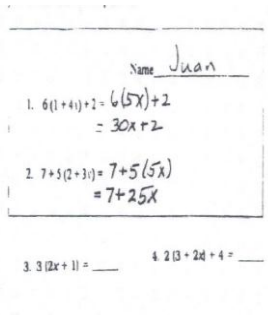
What is John's Error?



What is Ann's Error?



What is Juan's Error?



Interview data / Self-analysis

- When students are performing steps to solving an equation, have them perform think-alouds with you.
- Either write down their reasoning behind each step, or
- Have them write down their reasoning in a journal.
- Perform the interview before and after you implement the new form of instruction to determine if significant improvement is made.

Name the Most Common Answers

(Teacher Candidate study by Ryan & McCrae, 2005)

I) 0.3 x 0.24

a) 0.072
b) 0.08
c) 0.72
d) 0.8
e) 7.2

I) 0.3 x 0.24

Response	Inferred Misconception	Frequency
a) 0.072	CORRECT	36.1%
b) 0.08	0.3 is one-third or the decimal implies division	3.5%
c) 0.72	3x24 and adjust to 2 decimal places	41.1%
d) 0.8	0.3 is one-third or a decimal implies division and adjust to 1 decimal place	2.8%
e) 7.2	0.3 x 0.24 = 3 x 2.4	15.3%
OMITTED		1.4%

Remediation based on the error patterns of 0.3 x 0.24

- How could we teach based on the common error patterns?

X	0.2	0.04
0.3	0.06	0.012
0.06 + 0.012 = 0.072		

Name the Most Common Answers

(Ryan & McCrae, 2005)

II) $912 + {}^{4}/_{100}$ in decimal form

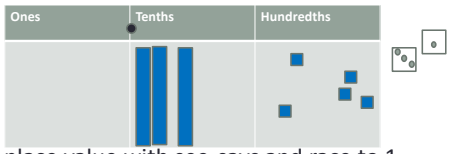
a) 912.4
b) 912.04
c) 912.004
d) 912.25
e) 912.025

II) $912 + {}^{4}/_{100}$ in decimal form

Response	Inferred Misconception	Frequency
a) 912.4	Hundredths is first decimal place	3.5%
b) 912.04	CORRECT	76.3%
c) 912.004	Onesths; Tenths, Hundredths	12.2%
d) 912.25	4/100 is ¼ or 100÷4 =1/25=0.25	6.0%
e) 912.025	100÷4=25 and onesths, tenths, and hundredths	1.6%
OMITTED		0.7%

Remediation based on the error patterns of $912 + {}^{4}/_{100}$ in decimal form

- How could we teach based on the common error patterns?



Teach place value with see-says and race to 1.

Name the Most Common Answers

(Ryan & McCrae, 2005)

III) 300.62 ÷ 100

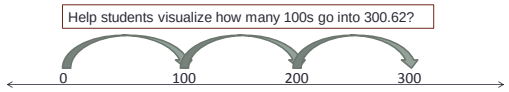
a) 30062
b) 30.062
c) 30.62
d) 3.0062
e) 3.62

III) 300.62 ÷ 100

Response	Inferred Misconception	Frequency
a) 30062	Move the decimal point 2 places to the right	0%
b) 30.062	Move the decimal point 1 place to the left	6.4%
c) 30.62	Cancel the zero	2.6%
d) 3.0062	CORRECT	68.8%
e) 3.62	Integer-decimal separation or cancel 2 zeros	22.0%
OMITTED		0%

Remediation based on the error patterns of 300.62 ÷ 100

- How could we teach based on the common error patterns?



Error Patterns with Fractions

Peter is adding with like denominators like this:
4 ¼ - 2 ¾ = 2 ½

What is he doing correctly?
What is he doing incorrectly?
What would be a good correction?

+4 + ¼	+3 $+^{5}/_{4}$	$+^{17}/_{4}$
- 2 - ¾	- 2 - ¾	$-^{11}/_{4}$
+2 $-^{2}/_{4}$	+1 $+^{2}/_{4}$	$^{6}/_{4}$
1 ½	1 ½	1 ½

(or) (or)

Number lines

- Show the following on a number line:
  - 3 + 2
  - 2 – 5
  - 3 x 2
  - 3 x 6
  - Identify $^{1}/_{3}$
  - Identify 1 ½
  - 2 ($^{2}/_{3}$)
  - ¾ ÷ ½

Common Algebra Errors (Dawkins, 2006)

- Where is the error made?

1. a=b	given
2. $ab=a^2$	multiply both sides by a
3. $ab-b^2=a^2-b^2$	subtract b^2 from both sides
4. b(a-b)=(a+b)(a-b)	factor both sides
5. b=a+b	divide both sides by a-b
6. b=2b	recall the given
7. 1=2	divide both sides by b

How do students tackle problems?

$\frac{3x - 6y - 9}{3}$

they might see the immediate possibility that:

$\frac{1\cancel{3}x - 2\cancel{6}y - 3\cancel{9}}{1\cancel{3}} = 1x - 2y - 3$

But what happens when computation isn't as straight forward?

$\frac{3x - 6y - 9}{x}$

they might see the immediate possibility that:

$\frac{\cancel{3x} - 6y - 9}{1\cancel{x}} = 3 - 6y - 9 = -6y - 6$

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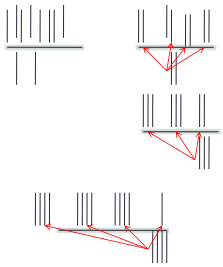
Connect Algebra to Arithmetic: Build on what they know

- 8 ÷ 2
- 8 ÷ 3
- 13 ÷ 4



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Significant Growth towards Algebra

Examined 6th and 7th grade preparedness towards Algebra according the Algebra Readiness Test

Study is limited (38 students with learning disabilities in mathematics; 2 schools in SC)

Alg Prep	Data/Prob	Equat	Decim	Expon	Fract	Comp	Graph	Integ
6th	no	no	no	minimal (ns)	no	minimal (ns)	no	no
7th	Significant Growth	no	no	minimal (ns)	minimal (ns)	no	no	no

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Algebra error patterns (parenthetical)

Dawkins, 2006

Math Concept	Correct	Incorrect
Square $4x = (4x)^2$	$(4x)^2 = (4)^2(x)^2 = 16x^2$	$4x^2$
Square -3	$(-3)^2 = (-3)(-3) = 9$	$-3^2 = -(3)(3) = -9$
Subtract 4x-5 from $x^2 + 3x - 5$	$x^2+3x-5 - (4x-5) = x^2+3x-5-4x+5 = x^2-x$	$x^2+3x-5-4x-5 = x^2-x-10$
Evaluate $-3\int 6x - 2dx$	$-3\int 6x - 2dx = -3(3x^2 - 2x) + c = -9x^2 + 6x + c$	$-3\int 6x - 2dx = -3 \cdot 3x^2 - 2x + c = -9x^2 - 2x + c$

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Trigonometry error patterns

Dawkins, 2006

Math Concept	Incorrect	Misconception
cos (x + y)	cos (x + y) = cos(x) + cos(y)	cos(x) is not multiplication
$\cos^{-1}x$	$\cos^{-1}x = 1/\cos(x)$	The -1 is not an exponent. Rather, it indicates we have an inverse trig function

This is why we explicitly state that n in $\sin^n x$ is a positive integer.

$\sin^n x = (\sin x)^n$

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Algebra expressions and equations

- Much of the confusion of algebra can be averted through a strong math background.
- However, the abstractness of variables and unknowns confuses many students.
- Hands-on instruction should be completed for concept understanding (e.g. graphical calculators) and task development (e.g. Multisensory Algebra)

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Arguments for and against algorithms

- Algorithmic instruction receives both admonishment and celebration, often by the same researcher.
- The most current argument against algorithms have been that they lead to blind adherence to stepwise rules without thought.

Improving on algorithms

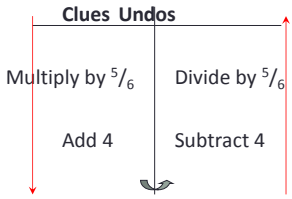
- Teach the process of the algorithm
- Allow students to interact with the procedures
- Oversee that the algorithm can cover future work that may appear similar to the current skill.
- Make the algorithm easy to remember
  - ex. PEMDAS; ROYGBIV

Reduce Meaningless Algorithms

- Students with memory concerns often receive remediation in the form of memory-based learning.
- For example, a child who can't learn the multiplication tables starts remediation with timed quizzes and flashcards. While this may be motivational, it is not necessarily instructional.

Clues and Undo's for procedures

$^5/_6X + 4 = 8$,
solve for X



Algebraic - The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions. (MA.D.1.3)
Operations - selects the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers, ratios, proportions, and percents, including the appropriate application of the algebraic order of operations. (MA.A.3.3)

Quadratic Equation example

$3x^2 + 10x - 8$

1. (A)(C) = (3)(-8) = -24
2. Factors of AC
   -6 & 4; -4 & 6; -8 & 3; -3 & 8; -12 & 2; -2 & 12
3. Sum or Difference to B
   -2 + 12 = 10
4. Substitute for B
   $3x^2 - 2x + 12x - 8$
5. Solve by groups
   x(3x–2) + 4(3x–2)
   (x+4) (3x-2)

1. AC
   (-8)(3) = -24
2. Factors that sum to B
   -8 + 3 = -5 ---NO
   +12 + -2 = +10 --- YES
2. Divide the factors by A
   12/3 = 4 and -2/3
3. Solve for X; x=-4 and x=2/3
4. Rewrite factors
   (x+4)(3x-2)

Mike Diamond,
SC math teacher

Let's Make Some Mnemonics and Acrostics

Quadratic

- Find out if the formula is quadratic
- Accurately compute to equal the whole number
- Compute the same numbers to equal the coefficient
- The parentheses come next
- Order what's in the parentheses
- Recheck the answer

Slope-Intercept

- Recognize and reorder the formula
- Identify the slope and intercept
- Set up the intercept
- Evaluate the slope
- Rise over run
- Use the dots to make a line
- Need to check your answer

Sample strategies

Identify the variables
Set up equations
Organize to balance
Let equations begin
Add variable side of equal sign
Total other side
Evaluate and check answer

Find what you are solving
Ask yourself about the parts
Set up the numbers
Tie down the sign

Discover the sign
Read the problem
Answer or draw or check
Write the answer

What algorithms can you turn into learning strategies?

Task Analyzing your Curriculum

1. Predict the optimum sequence to reach the outcome your textbook's chapter before you begin teaching
2. Match your task analysis to the textbook
3. Note commonalities and differences
4. Check earlier chapters to see if they cover the differences. Check later chapters to see if they cover the differences.
5. Check supplemental guides to see if they cover the differences
6. Develop additional instruction to complement the current text / curriculum
7. Sequence the instruction as your students need

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