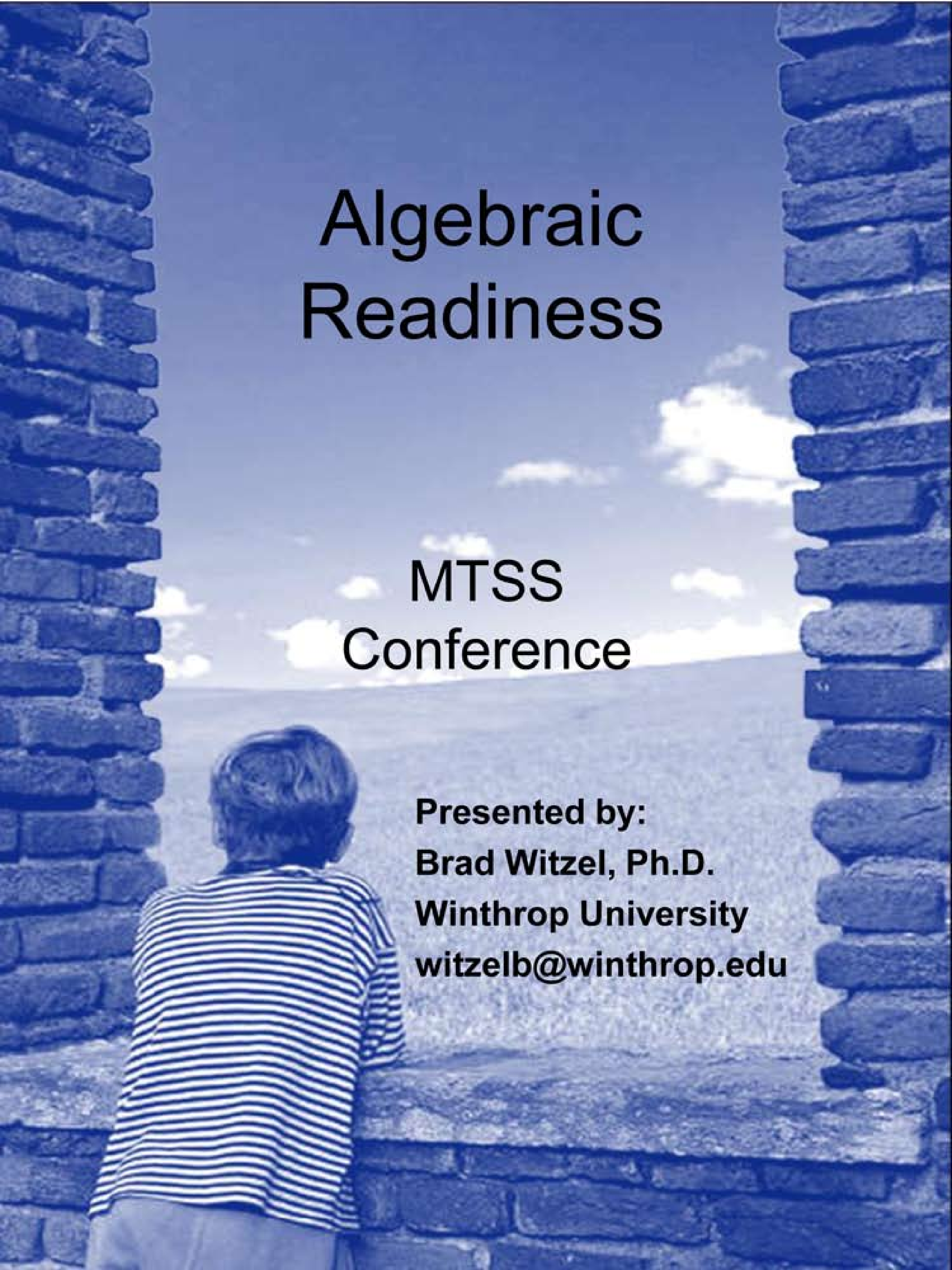


Algebraic Readiness

MTSS Conference

Presented by:
Brad Witzel, Ph.D.
Winthrop University
witzelb@winthrop.edu

Algebraic Readiness

Bradley Witzel, Ph.D.
Winthrop University
witzelb@winthrop.edu
803-323-2453
Twitter @BradWitzel

What Math Knowledge is Needed to Solve these Equations?

$2x + 5 = 18$
$\quad - 5 \quad - 5$
$2x \quad = 13$

$\frac{2x}{2} = \frac{13}{2}$

$1x \quad = 6\,½$

$(y - 5)(y + 2)$

$(y)(y) + (y)(2) - (5)(y) - (5)(2)$

$y^2 + 2y - 5y - 10$

$y^2 - 3y - 10$

U.S. Math Performance

The 2011 National Assessment of Educational Progress (NAEP) reported US math achievement as:
- 18% of Grade 4 students scored below the basic level
- 28% of Grade 8 students scored below the basic level

*Grade 12 students scores could not be compared

2011 NAEP data revealed that:
- 10% of 4th grade KS students scored below basic
- 20% of 8th grade KS students scored below basic

(White 14%; Hispanic 35%; African-American 41%)

U.S. Math Performance
Students with Disabilities

2011 NAEP report on students with disabilities
National percent scoring below basic
- 45% of Grade 4
- 65% of Grade 8

Kansas
- 34% of 4th grade
- 57% of 8th grade

U.S. Math Performance
Students with Disabilities

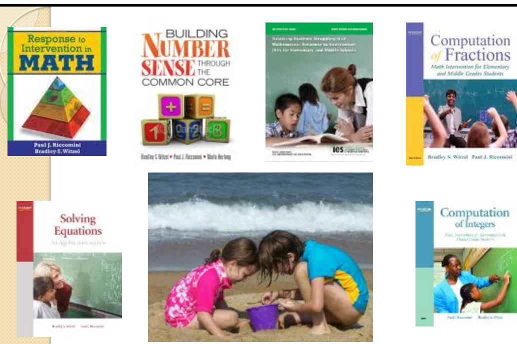
2011 NAEP report on ELL population
National percent scoring below basic
- 42% of Grade 4
- 72% of Grade 8

Kansas
- 17% of 4th grade
- 50% of 8th grade

Mathematics Performance

Translated to Real World Performance
- 78% of adults cannot explain how to compute interest paid on a loan
- 71% cannot calculate miles per gallon
- 58% cannot calculate a 10% tip
- 27% of 8th graders could not correctly shade 1/3 of a rectangle
- 45% could not solve a word problem that required dividing fractions

Mathematics Advisory Panel Final Report, 2008



I hope that you leave affected.

CCSS Domains

Grade Level Domains	High School Conceptual Categories
K – 5 • Counting and Cardinality • Operations and Algebraic Thinking • Number and Operations in Base Ten • Number and Operations – Fractions • Measurement and Data • Geometry **6-8** • Ratios and Proportional Relationships • The Number System • Expressions and Equations • Functions • Geometry • Statistics and Probability	• Number and Quantity • Algebra • Functions • Modeling • Geometry • Statistics and Probability

From RI CCSS presentation

Growth in MS towards Algebra?

Examined 6th and 7th grade preparedness towards Algebra according the Algebra Readiness Test

Study is limited (38 students with learning disabilities in mathematics; 2 schools in SC)

Alg Prep	Data/Prob	Equat	Decim	Expon	Fract	Comp	Graph	Integ
6th	no	no	no	minimal (ns)	no	minimal (ns)	no	No
7th	Significant Growth	no	no	minimal (ns)	minimal (ns)	no	no	no

Teaching beyond grade and course

- Your responsibility for what students learn in your course implies that you are responsible for what they learned before your course.
- Students have been introduced and presented a lot of math information. However, every year teachers blame teachers from the year before for not preparing their students.
- Not only must you know your own course content, you must be aware of previous grade level content and what is expected in the following year.

What we are doing today:

- Learning the necessary foundational skills for algebra success
- Reviewing end of grade / course expectations
- Instructional supports
- Vertical and Horizontal Planning

NMP quotes

- "Few curricula in the United States provide sufficient practice to ensure fast and efficient solving of basic fact combinations and execution of the standard algorithms" (p. 26).
- "…students should be able to proceed successfully at least through the content of Algebra II…" (p. 15)
- "Teachers should recognize that from early childhood through elementary school years, the spatial visualization skills needed for learning geometry have already begun to develop. In contrast to the claims of Piagetian theory, young children appear to possess at least an implicit understanding of basic facts of Euclidean concepts. However formal instruction is necessary to ensure that children build upon this knowledge to learn geometry" (p. 29)

More NMP quotes

- "Differences in teachers account for 12% to 14% of total variability in students' mathematics achievement gains" (p. 35).
- "Calculators should not be used on test items designed to assess computational facility" (p. 61).
- "Publishers should make every effort to produce much shorter and more focused textbooks" (p. xxiv).
- Paraphrased - Students need clear models with think alouds, many examples and opportunities for practice, and frequent feedback. (p. 48)
- More rigorous research for this group of students is needed (p. 49).

©Witzel, 2012 13

A new focus within the CCSSM

1. More Instructional Time: fewer topics covered in greater depth
2. Planned Progressions: instruction is connected within and across grades
3. Proficiency: perform mathematics procedures with speed and accuracy
4. Application: applying math to solve a problem
5. Balanced Learning: achieve fluency and conceptual understanding

©Witzel, 2012 14

What should be covered before formal algebra? (Gersten, Clarke, & Witzel, 2008)

- Fluency with standard algorithms
- Understanding properties
  - Commutative
  - Associative
  - Distributive
- Basic measurement concepts and operations of 2 and 3 dimensional obejcts
- Word problem translations into symbols

©Witzel, 2012 15

Algebra and CCSS – just a reminder

8th Grade

The Number System
- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations
- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions
- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry
- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability
- Investigate patterns of association in bivariate data.

Algebra

Seeing Structure in Expressions
- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Expressions
- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

Creating Equations
- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities
- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

©Witzel, 2012 16

Algebra Teachers (NMP, 2008)

743 algebra teachers in 310 schools nationally responded to a survey on algebra instruction and student learning in 2007.

Findings:

- The teachers generally rated their students' background preparation for Algebra I as weak. The three skill areas in which teachers reported their students have the poorest preparation are rational numbers, word problems, and study habits
- Regarding the best means of preparing students, 578 suggested a greater focus on mastery of elementary mathematical concepts and skills
- Teachers were less excited about how current textbook approaches meet the needs of diverse student populations

©Witzel, 2011 17

Arithmetic to Algebra Gap

(Witzel, Smith, & Brownell, 2001)

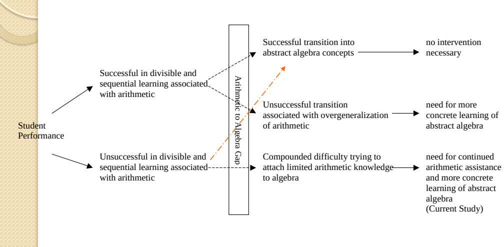


Figure 1. Flowchart of Algebraic Needs for Students Who Experience Difficulty in Math

©Witzel, 2004 18

3rd grade item on place value

The digits in a three-digit number represent the amounts of hundreds, tens, and ones. Fill in the chart to show the amounts of hundreds, tens, and ones in the number 523.

Number	Hundreds	Tens	Ones
523			

In the box below, write a number that meets the following conditions.

- The number must be between 1 and 9.
- When the number is subtracted from 523, the digit in the ones place of the difference is **greater** than the ones place of 523.

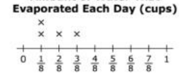
CCSS

4th grade - fractions

Judy conducted an experiment. She put a total of $2\frac{1}{8}$ cups of water into an empty container. Then, Judy recorded the amount of water that evaporated from the container each day for four days.

The line plot below shows the amount of water that evaporated from the container on each of the four days.

Amount of Water That Evaporated Each Day (cups)



Each × represents 1 day.

What mixed number represents the amount of water left in the container at the end of the fourth day?

Cups

CCSS

4th grade – deciphering a table

A scientist watched a group of squirrels collect acorns. Each squirrel **ate** some of the collected acorns and **stored** the rest of the collected acorns.

The table below shows data for three squirrels in the group. The number of acorns each squirrel **stored** is missing from the table. Fill in the data that are missing from the table.

Acorns Collected by Squirrels

Squirrel	Number Eaten	Number Stored	Total Number Collected
X	40		100
Y	50		105
Z	35		95

CCSS

5th grade – volume and measurement

Shelbi needs wooden boards to build a platform. Each board is shaped like a rectangular prism and has a length of 2 meters, a height of 40 millimeters, and a width of 100 millimeters, as shown below.

To build the platform, Shelbi will place the boards side by side, as shown in this diagram. The platform will have a total width of 12 meters.

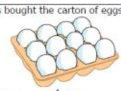
What is the **least** number of boards that Shelbi needs to build the platform?

Boards

CCSS

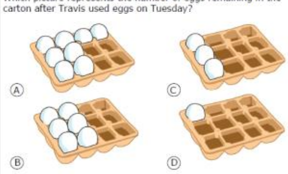
5th grade – fraction representation

On Sunday, Travis bought the carton of eggs pictured below.



- On Monday, Travis used $\frac{1}{4}$ of the eggs in the carton.
- On Tuesday, Travis used $\frac{2}{3}$ of the eggs that **remained** in the carton after Monday.

Which picture represents the number of eggs remaining in the carton after Travis used eggs on Tuesday?



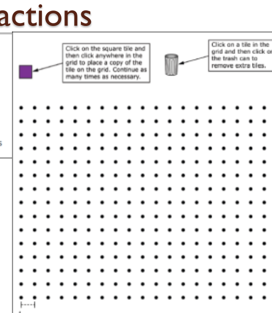
CCSS

5th grade - fractions

Gregory is installing tile on a rectangular floor.

- He is using square tiles.
- The length of a side of each tile is $\frac{1}{2}$ foot.
- The area of the floor is 22 square feet.
- The width of the floor is 4 feet.

Use the grid to model the rectangular floor that Gregory is installing.



What is the **length**, in feet, of the floor?

CCSS

6^{th} grade – ratio and proportions

Ben's Game World is having a sale on video games. The store is offering a sale pack of 4 video games for $43.80. What is the unit price of a video game in the sale pack?

$ ☐

Roberto's Electronics is also having a sale on video games. The unit price of any video game at Roberto's Electronics is the same as the unit price of a video game in the sale pack at Ben's Game World. How much would it cost a customer for 7 video games at Roberto's Electronics?

$ ☐

CCSS

6^{th} grade – ratio and proportion

In art class, Marvin painted tiles to use for a project. For every 5 tiles he painted blue, he painted 8 tiles green.

Identify the equivalent ratio(s) of blue tiles to green tiles. Select all that apply.

Ⓐ 20:23
Ⓑ 40:25
Ⓒ 50:800
Ⓓ 60:96

CCSS

6^{th} grade – fractions computation

Hisaki is making sugar cookies for a school bake sale. He has $3\frac{1}{2}$ cups of sugar. The recipe calls for $\frac{3}{4}$ cup of sugar for one batch of cookies. Which equation can be used to find b, the total number of batches of sugar cookies Hisaki can make?

Ⓐ $3\frac{1}{2} \times \frac{3}{4} = b$
Ⓑ $3\frac{1}{2} \div \frac{3}{4} = b$
Ⓒ $3\frac{1}{2} + b = \frac{3}{4}$
Ⓓ $3\frac{1}{2} - b = \frac{3}{4}$

CCSS

7^{th} grade – fractions and proportions

Roberto is making cakes. The number of cups of flour he uses is proportional to the number of cakes he makes.

Roberto uses $22\frac{1}{2}$ cups of flour to make 10 cakes.

Which equation represents the relationship between f, the number of cups of flour Roberto uses, and c, the number of cakes he makes?

Ⓐ $f = \frac{4}{9}c$
Ⓑ $f = 2\frac{1}{4}c$
Ⓒ $f = 2\frac{1}{2}c$
Ⓓ $f = 10c$

CCSS

7^{th} grade - computation

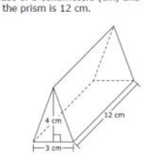
Identify the number(s) that makes each statement true. You may select more than one number for each statement.

1a. $-4.8 + \square =$ a positive number ○ -5.2 ○ 4.9
1b. $\square - 1\frac{1}{2} =$ a negative number ○ $\frac{3}{2}$ ○ $-\frac{7}{3}$
1c. $\square + 5 =$ zero ○ -5 ○ 5
1d. $-2.15 - \square =$ a negative number ○ -1.75 ○ 1.34

CCSS

7^{th} grade - geometry

Look at the triangular prism below. Each triangular face of the prism has a base of 3 centimeters (cm) and a height of 4 cm. The length of the prism is 12 cm.



What is the volume, in cm^3, of this triangular prism?

☐ cm^3

CCSS

8th grade – radicals and integers

The average distance from Jupiter to the Sun is about 5×10^8 miles. The average distance from Venus to the Sun is about 7×10^7 miles.

The average distance from Jupiter to the Sun is about how many times as great as the average distance from Venus to the Sun?

______ times

CCSS

8th grade –radicals and integers

Select **all** of the expressions that have a value between 0 and 1.

- (A) $8^7 \cdot 8^{-12}$
- (B) $\frac{7^4}{7^{-3}}$
- (C) $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^9$
- (D) $\frac{(-5)^6}{(-5)^{10}}$

CCSS

8th grade - equations

Three students solved the equation $3(5x-14)=18$ in different ways, but each student arrived at the correct answer. Select **all** of the solutions that show a correct method for solving the equation.

(A)
$$\begin{aligned} 3(5x-14)&=18\\ 8x-14&=18\\ +14\quad&\quad+14\\ \frac{8x}{8}&=\frac{32}{8}\\ x&=4 \end{aligned}$$

(B)
$$\begin{aligned} \frac{1}{\cancel{3}}\cdot\cancel{3}(5x-14)&=18\cdot\frac{1}{3}\\ 5x-14&=6\\ +14\quad&\quad+14\\ \frac{5x}{5}&=\frac{20}{5}\\ x&=4 \end{aligned}$$

(C)
$$\begin{aligned} 3(5x-14)&=18\\ \frac{15x}{15}-\frac{42}{15}&=\frac{18}{15}\\ +\frac{42}{15}\quad&\quad+\frac{42}{15}\\ x&=\frac{60}{15}\\ x&=4 \end{aligned}$$

CCSS

8th grade – radicals

Classify the numbers in the box as perfect squares and perfect cubes. To classify a number, drag it to the appropriate column in the chart. Numbers that are neither perfect squares nor perfect cubes should **not** be placed in the chart.

1 64 96 125 200 256 333 361

Perfect Squares but Not Perfect Cubes	Both Perfect Squares and Perfect Cubes	Perfect Cubes but Not Perfect Squares

CCSS

What difficulties will stand in the way of answering these questions?

1)
2)
3)
4)
5)
6)
7)

Do middle school math courses add up to algebra preparation?

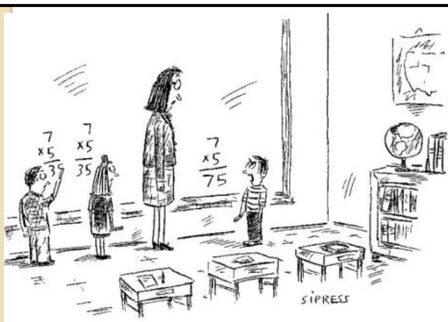
Sanders, Riccomini, & Witzel, 2005

Code	Category	Entering Math Tech 1	Entering Algebra 1
DAPR	Data Analysis, Probability & Statistics	39 (46.4%)	85 (88.5%)
DECM	Decimals, their Operations and Applications: Percent	11 (13.1%)	64 (66.7%)
EQTN	Simple Equations and Operations with Literal Symbols	35 (41.7%)	80 (83.3%)
EXPS	Exponents and Square Roots; Scientific Notation	27 (21.1%)	62 (64.6%)
FRAC	Fractions and their Applications	3 (3.6%)	43 (44.8%)
GMMS	Measurement of Geometrical Objects	20 (23.8%)	56 (58.3%)
GRPH	Graphical Representation	13 (15.5%)	59 (61.5%)
INTG	Integers, their Operations & Applications	27 (32.1%)	83 (86.5%)
	Total Number of Students per course	84	96

So, Where are we exactly?

What constitutes good instruction for struggling students (Gersten, Chard, & Witzel, 2008)

- Model approaches to solving problems many times – both easy and difficult
- "Think aloud" and teach students to do the same
- Frequent practice
- Independent practice should include discriminatory problems
- Teach word problems and computation
- Use visuals, such as CRA, to represent problems



"It may be wrong, but it's how I feel."

K-2 Objectives

a) numeral and number
b) magnitude comparisons
c) counting strategies
d) computation
e) fact accuracy and fluency

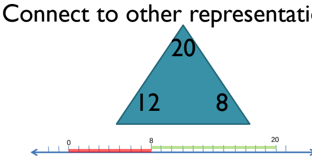
Themes:
Manipulatives as a level of learning
Assessment throughout

Assessment of Number Sense Components

- Number or Numeral Recognition (Baker et al., 2002; Jordan et al., 2008; Seethaler & Fuchs, 2010)
- Magnitude Comparisons (Chard et al., 2005; Clarke et al., 2008; Seethaler & Fuchs, 2010)
- Counting Principles (Clarke et al., 2008; Lembke & Foegen, 2009; Methe, Hintze, & Floyd, 2008)
- Fact Fluency (Bryant et al., 2008)
- Word Problems (Locuniak & Jordan, 2008)

Number Lines

Use number lines physically and pictorially
Connect to other representations



Place Value

- "Any concept dependent on number is dependent on place value" (Sharma, 1993).
- Place value is hard to assess because of its involvement in other math processes and skills
- Common Core
  - K-Working with numbers 11-19 to gain foundations for place value
  - 1-Understand a two-digit number represents amounts of tens and ones
  - 2-Three-digit numbers recognition
  - 3-multi-digit arithmetic

Examples from the CCSS curriculum document

CC.3.NBT.2 Use place value understanding and properties of operations to perform multi-digit arithmetic. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (A range of algorithms may be used.)

Place Value skills

- 27 = 2 tens and 7 ones
- 45 = 4 tens and 5 ones

- Should be represented physically and verbally
- Advanced learners should use place value within a calculation exercise.

Benefits of place value knowledge

13
-7

10 + 3
-7
3 + 3 = 6

341
-196

300 + 40 + 1
-100 – 90 – 6

200 + 130 + 11
- 100 – 90 – 6
100 + 40 + 5 = 145

X	50	3
20	1000	60
8	400	24
1000+400+60+24 = 1484		

Extensions of place value

- Use expanded notation and/or arrays with these problems

1) 45+23
2) 38+14
3) 57-31
4) 34-18
5) 2.13+3.52
6) 4.37-1.27
7) 6.22-3.45
8) 57x29
9) 25x14
10) 3.6x2.8

Focus on the facts (Parkhurst et al, 2010, p. 111)

- "Students who can complete basic math computations problems with rapidity are likely to expend less time and effort on math activities and have less math anxiety"
- "Consequently, those with greater basic-fact fluency are more likely to choose to engage in math activities, which further enhance skills."

Common Core

- CC.3.OA.7 Multiply and divide within 100. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. **By the end of Grade 3, know from memory all products of one-digit numbers.**

Overloading cognition

"Without the ability to retrieve facts directly or automatically, students are likely to experience a high cognitive load as they perform a range of complex tasks" (Woodward, 2006, p. 269).

Find new ways to reach the students

IES Practice Guide on Fractions

(Siegler et al., 2010)

http://ies.ed.gov/ncee/wwc/pdf/practiceguides/fractions_pg_093010.pdf

Recommendation 1. Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts

Recommendation 2. Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward

Recommendation 3. Help students understand why procedures for computations with fractions make sense

Recommendation 4. Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems

Recommendation 5. Professional development programs should place a high priority on improving teachers' understanding of fractions and of how to teach them

Horizontal Planning: A grade level's success is a team effort

- **Set up grade level team non-negotiables**
- Key math skills need to be understood by all students.
- Use the CCSS to determine which math skills are benchmarks
- Within the CCSS for math, "the standard algorithm" is used four times. What will be your grade level team's standard algorithm?

Horizontal Planning Activity

- List the steps to multiplication of multi-digit numbers

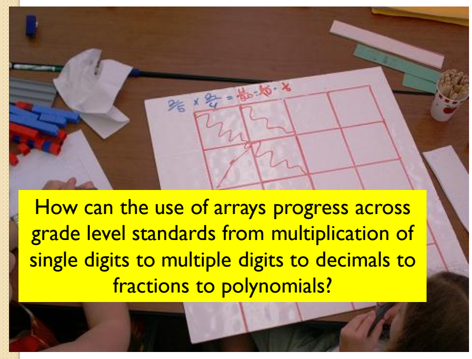
Ex. 24 x 76 = or 24
<u>x76</u>

CCSS 4.NBT.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

- Compare approaches across same grade teachers.
- What does this mean to mathematics instruction at your grade level?

Vertical Planning: One teacher's success depends on the previous teacher's

- The success of each grade level builds upon the next.
- Key math skills need to be understood by all students. To help students grow in math, those key skills can be built across grade levels.
- Within the CCSS for math, "the standard algorithm" is used four time across three grade levels.
- Build progressions to relate each grade's standards to the next.



An example of an algorithmic progression using CCSS

- Second grade "Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends."

4 + 4 + 4 (teaching multiples)

			4
			8
			12

- Third grade "Multiplication and division within 100"
- 8 x 6 = ?

6

x	x	x	x	x
x	x	x	x	x
x	x	x	x	x
x	x	x	x	x
x	x	x	x	x
x	x	x	x	x
x	x	x	x	x
x	x	x	x	x

- Fourth grade "Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models."
- 24x76= ?

multiply	70	6
20	1400	120
4	280	24

1400+120+280+24 = 1824

- Fourth grade "Apply and extend previous understandings of multiplication to multiply a fraction by a whole number."
- 2 x ¾

multiply	$^{3}/_{4ths}$
2	$^{6}/_{4ths}$

(2x3) / 4 = 6/4

- Fifth grade "Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used."
- 7.6 x 2.4 = ?

multiply	7	.6
2	14	1.2
.4	2.8	.24

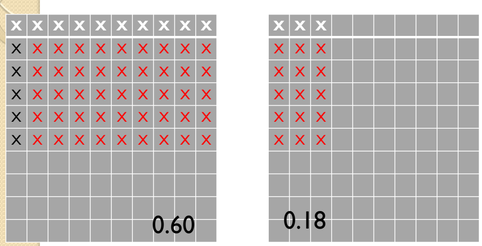
14+1.2+2.8+0.24 = 18.24

- Fifth grade "Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas."
- 2 1/3 x 4 1/2

multiply	2	1/3
4	8	4/3
1/2	2/2	1/6

Using Arrays for Decimals

(1.3)(0.6)



Total = 0.78

How Arrays Translate

$3\ ^1/_2$
$\underline{\times 1\ ^2/_3}$

5.3
$\underline{\times 2.4}$

x	3	1/2
1	3	1/2
2/3	6/3	2/6

$^3/_1 + {}^6/_3 + {}^1/_2 + {}^2/_6 =$

$^{18}/_6 + {}^{12}/_6 + {}^3/_6 + {}^2/_6 = {}^{35}/_6$

$= 5\ ^5/_6$

x	5	0.3
2	10	0.6
0.4	2.0	0.12

10 + 0.6 + 2.0 + 0.12 = **12.72**

- Algebra "Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials."
- (3x – 1)(4x + 5)

multiply	3x	-1
4x	$12x^2$	-4x
+5	15x	-5

$12x^2 - 4x + 15x - 5 = 12x^2 + 11x - 5$

Next Steps

- What areas of math require concentrated effort to achieve in math?
- What should be horizontally planned?
- How do we vertically plan instruction?

Conclusion

"A man is like a fraction whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator, the smaller the fraction."

Leo Tolstoy

References A

Cawley, J. F., Parmar, R. S., Lucas-Fusco, L. M., Kilian, J. D., & Foley, T. E. (2007). Place value and mathematics for students with mild disabilities: Data and suggested practices. *Learning Disabilities: A Contemporary Journal, 15*(1), 21-39.

Clements, D. H. (2004). *Major themes and recommendations*. In D. H. Clements, J. Sarama, & A. M. DiBiase (Eds.), Engaging young children in mathematics: Standards for early childhood mathematics education. Mahwah, NJ: Lawrence Erlbaum Associates.

Fuson, K. C., & Briars, D. (1990). Using a base-ten blocks learning/teaching approach for first- and second-grade place-value and multidigit addition and subtraction. *Journal for Research in Mathematics Education, 21*, 180 – 206.

Gersten, R., Clarke, B. S., & Jordan, N. C. (2007). *Screening for mathematics difficulties in K-3 students.* Portsmouth, NH: RMC Research Corporation, Center on Instruction.

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development, 74*(3), 834-850.

Long, V. (2009). Adding "place" value to your mathematics instruction. *Connect: Innovations in K-8 Science, Math, and Technology, 23*(1), 10-12

Mann, A., Moeller, K., Pixner, S., Kaufmann, L., & Nuerk, H-C. (2011). Attentional strategies in place value integration: A longitudinal study on two-digit number comparison. *Journal of Psychology, 219*(1), 42–49.

Roche, A. (2010). Decimats: Helping student to make sense of decimal place value. *Australian Primary Mathematics Classroom, 15*(1), 4-10.

Saxton, M., & Cakir, K. (2006). Counting-on, trading, and partitioning: Effects of training and –prior knowledge on performance on base-10 tasks. *Child Development, 77*(3), 767-785.

Schmidt, M. E. (1995). Mathematics intervention: Second grade place value concepts. *Education, 116*(2), 229-234.

Sharma, M. C. (1993). Place value concept: How children learn it and how to teach it. *Math Notebook, 10*(1&2), 1-23.

Thompson, I. (2009). Place value? *Mathematics Teaching, 215*, 4-5.

Toppo, G. (2008, September 22). Study: Many 8th-graders can't handle algebra. *USA Today*.

Witzel, B. S. (in-review). Place value is more than number position.

Witzel, B. S., Riccomini, P. J., & Larson, M. (in-press). *Number sense instruction and intervention within the common core.* Thousand Oaks, CA: Corwin Press.

Uy, F (2003). The Chinese numeration system and place value. *Teaching Children Mathematics, 9*(5), 43-47.

References B

Butler, F. M., Miller, S. P., Crehan, K., Babbit, B., & Pierce, T. (2003). Fraction instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research & Practice, 18*(2), 99-111.

Gagnon, J. G., & Maccini, P. (2007). Teacher-reported use of empirically validated standards-based instructional approaches in secondary mathematics. *Remedial and Special Education, 28*(1), 43-56.

Hutchinson, N. L. (1993). Second invited response: Students with disabilities and mathematics education reform-Let the dialog begin. *Remedial and Special Education, 14*(6), 20-23.

Jordan, L., Miller, M. D., & Mercer, C. D. (1999). The effects of concrete to semi-concrete to abstract instruction in the acquisition and retention of fraction concepts and skills. *Learning Disabilities: A Multidisciplinary Journal*, 9, 115-122.

Maccini, P., & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research & Practice, 15*, 10-21.

Maccini, P., Mulcahy, C. A., & Wilson, M. G. (2007). A follow-up of mathematics interventions for secondary students with learning disabilities. *Learning Disabilities Research & Practice, 22*(1), 58-74.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Parkhurst, J., Skinner, C. H., Yaw, J., Poncy, B., Adcock, W., & Luna, E. (2010). Efficient class-wide remediation: using technology to identify idiosyncratic math facts for additional automaticity drill. International Journal of Behavioral Consultation and Therapy, 6 (2), p111-123.

Witzel, B. (2005). Using CRA to teach algebra to students with math difficulties in inclusive settings. *Learning Disabilities: A Contemporary Journal*, 3(2), 49-60.

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice*, 18(2), 121-131.

Witzel, B. S., & Riccomini, P. J. (2009). *Computation of fractions*. Upper Saddle River, NJ: Pearson.