

**RTI - Mathematics: What do we know and where do we go from here?**

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**Increasing recognition of the importance of mathematical knowledge**

- ❖ “For people to participate fully in society, they must know basic mathematics. Citizens who cannot reason mathematically are cut off from whole realms of human endeavor. Innumeracy deprives them not only of opportunity but also of competence in everyday tasks”. (Adding it Up, 2001)

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**State of Mathematics**

- ❖ Achievement on the NAEP trending upward for 4th/8th grade and steady for 12th grade
  - ❖ Large numbers of students still lacking proficient skills
  - ❖ Persistent income and ethnicity gaps
  - ❖ Drop in achievement at the time algebra instruction begins
- ❖ TIMS data indicate significant lower levels of achievement between US and other nations
  - ❖ Gap increase over time
- ❖ Jobs requiring intensive mathematics and science knowledge will outpace job growth 3:1 (STEM) and everyday work will require greater mathematical understanding

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**High Level of Interest in Mathematics Achievement**

- ❖ National Mathematics Advisory Report
  
- ❖ National Council Teachers of Mathematics: Focal Points
  
- ❖ National Research Council: Adding it Up

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**Response to Intervention**

- ❖ Reauthorization of IDEA (2004) allowed for RTI to be included as a component in special education evaluations
- ❖ Premised on the use of research based interventions and student response to intervention
  - ❖ Students who respond are not identified as learning disabled
  - ❖ Students who do not respond are referred for a complete evaluation and potential identification as learning disabled

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**Started to serve individual students but implications for all students and teachers**

- ❖ Linked closely to an early identification and prevention model of delivery
- ❖ Provides for the delivery of tiered services across traditional boundaries (e.g.Special and General Education)
- ❖ Most often implemented by schools using a schoolwide model of instruction.....
  - ❖ In Reading, but what about Math!

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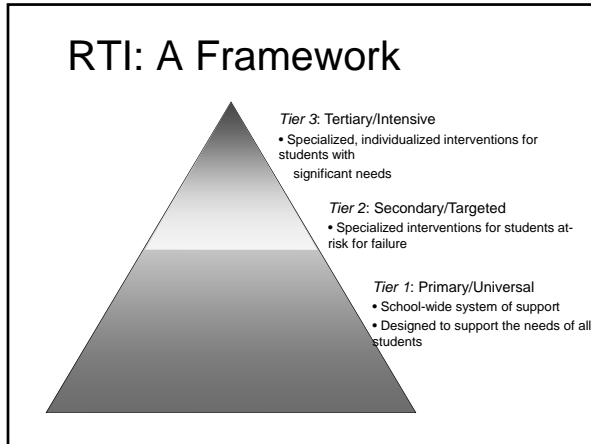
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### What is Needed to start “RTI”

- ❖ **Primary:**
  - ❖ Valid system for screening
  - ❖ An array of evidence-base intervention or at least promising interventions for Tier 2 students
- ❖ **Secondary:**
  - ❖ System for progress monitoring
  - ❖ Core instructional program
  - ❖ Diagnostic assessments

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### Discussion

- ❖ What parts do you currently have in place?
- ❖ What are you needing to add?

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**Assisting Students Struggling with Mathematics: Response to Intervention for Elementary and Middle Schools**

The report is available on the IES website:

<http://ies.ed.gov/ncee> &  
<http://ies.ed.gov/ncee/wwc/publications/practiceguides/>

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**Panelists**

- ❖ Russell Gersten (Chair)
- ❖ Sybilla Beckman
- ❖ Ben Clarke
- ❖ Anne Foegen
- ❖ Laurel Marsh
- ❖ Jon R. Star
- ❖ Bradley Witzel

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**Issues considered by the Panel**

- ❖ Paucity of Research
  - ❖ A lit search for studies on reading disabilities studies and math disability studies from 1996-2005 found over 600 studies in the area of reading and less than 50 for mathematics (14:1)
- ❖ Different models (i.e. number of tiers)
- ❖ Different approaches
  - ❖ Individualized problem solving
  - ❖ Standard protocol

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### The Recommendations by Tier

- ❖ Tier 1
  - ❖ Universal Screening
- ❖ Tier 2 and Tier 3
  - ❖ Focus instruction on whole number for grades k-5 and rational number for grades 6-8.
  - ❖ Explicit and systematic instruction
  - ❖ Solving word problems
  - ❖ Use of Visual Representations
  - ❖ Building fluency with basic arithmetic facts
  - ❖ Progress monitoring
  - ❖ Use of motivational strategies

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### Tier 1: Components

- ❖ Universal Screening
  
- ❖ Core Curriculum based on expert judgment
  
- ❖ Supplements to the core curriculum

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### Recommendation 1

Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.

- ❖ Level of Evidence: **Moderate**

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### Suggestions

- ❖ Have a building level team select measures based on critical criteria such as reliability, validity and efficiency.
- ❖ Select screening measures based on the content they cover with a emphasis on critical instructional objectives for each grade level.
- ❖ In grades 4-8, use screening measures in combination with state testing data.
- ❖ Use the same screening tool across a district to enable analyzing results across schools

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### Screening

1. Measures used in screening vary from short duration fluency measures to in-depth measures.
  - ❖ Short duration: Early Numeracy CBM; CBM computational and conceptual probes.
  - ❖ Number Knowledge Test (approx. 15 min).
  - ❖ Math specific tests such as TEMA, Key Math - also used in diagnostic testing.
2. Goal is to make accurate predictions about who needs and who does not need additional services.
3. Must balance efficiency of screening process with goal of accurate predictions.

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### Measures for Screening

- ❖ Early Grades
  - ❖ Short duration fluency measures.
    - ❖ E.g. Early Magnitude Comparison
    - ❖ Missing Number in a series (strategic counting) e.g.. 7, 9..... X, 10, 11
  - ❖ Robust Indicators..... But only for one year.
  - ❖ For long term prediction: working memory, PA measures show promise (E.g. reverse digit span)...

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### Upper Grades Screening

- ❖ Algebra measures
  - ❖ Designed by Foegen and colleagues assess pre-algebra and basic algebra skills.
  - Administered and scored similar to Math-CBM
  
- ❖ Math CBM Computation and Concepts and Applications
  - ❖ Concepts and Applications showed greater validity in 6th, 7th, and 8th grade

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### Core Curricula - Less is More

- ❖ US curricula tend to cover more topics with less depth resulting in persistent review across grades versus closure after exposure, development and refinement.
  
- ❖ NCTM Focal Points to the Common Core
  - ❖ Emphasize critical topics at each grade level
  - ❖ (e.g. 2nd grade)

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### Core curriculum

- ❖ National Math Panel
  - ❖ Need to develop understanding and mastery of
    - ❖ Whole number: understand place value, compose/decompose numbers, meaning of operations, algorithms and automaticity with facts, apply to problem solving, use/knowledge of commutative, associative, and distributive properties,
    - ❖ Rational number: locate +/- fractions on number line, represent/compare fractions, decimals percents, sums, differences products and quotients of fractions are fractions, understand relationship between fractions, decimals, and percents, understand fractions as rates, proportionality, and probability, computational facility
    - ❖ Critical aspects of geometry and measurement: similar triangles, slope of straight line/linear functions, analyze properties of two and three dimensional shapes and determine perimeter, area, volume, and surface area

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**Example: Tier 1 Core (Early Learning in Mathematics - ELM)**

❖ Structure

- ❖ 120 Core Lessons divided into 4 quarters
  - ❖ 30 minutes whole class instruction
  - ❖ 15 minutes teacher directed written work
  - ❖ End of quarter assessment of progress

❖ Daily Calendar Lessons / Activities

- ❖ 15 minutes daily, whole class "circle" time
- ❖ Monthly booklets with objectives and application activities

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**Example: Tier 1 Core (ELM)**

- ❖ National Math Advisory Panel (2008) recommends a focused, coherent progression of mathematics learning with emphasis on proficiency with key topics
- ❖ ELM focuses on key strands rather than a broad array of mathematical content
  - ❖ Numbers and Operations
  - ❖ Geometry
  - ❖ Measurement
  - ❖ Vocabulary (NCTM Process Standard, 2000)




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**Example: Tier 1 supplemental intervention (STEEP)**

- ❖ Entire class is screened on a computation probe
- ❖ If class is below criterion established by Deno, then entire class receives Tier 1 intervention (i.e. practice in computation and facts for 10 min daily for 10 days)
- ❖ Students who do not respond to the Tier 1 intervention are provided a similar Tier 2 intervention consisting of peer tutoring on computation problems
- ❖ Limited scope and duration of the Tier 1 intervention

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### **Tier 2 and 3 Components**

- ❖ Content of the Intervention
- ❖ Instructional design of the Intervention
- ❖ Progress monitoring and diagnostic assessments

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### **Recommendation 2**

Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 3. These materials should be selected by committee.

- ❖ Level of Evidence: **Low**

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### **Suggestions**

- ❖ For tier 2 and 3 students in grades k-5, interventions should focus on the properties of whole number and operations. Some older students would also benefit from this approach.
- ❖ For tier 2 and 3 students in grades 4-8, interventions should focus on in depth coverage of rational number and advanced topics in whole number (e.g. long division).

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### Suggestions

- ❖ Districts should appoint committees with experts in mathematics instruction and mathematicians to ensure specific criteria are covered in-depth in adopted curriculums.
- ❖ Integrate computation with problem solving and pictorial representations
- ❖ Stress reasoning underlying calculation methods
- ❖ Build algorithmic proficiency
- ❖ Contain frequent review of mathematical principles
- ❖ Contain assessments to appropriately place students in the program

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### Intervention Content

- ❖ Wu and Milgram (CA standards) for at-risk 4th to 7th grade students
- ❖ Recommend 2 hours of instruction per day
- ❖ Taught by teacher with content knowledge expertise
- ❖ Topics
  - ❖ Place Value and Basic Number Skills (1st-3rd grade skills)
  - ❖ Fractions and Decimals
  - ❖ Ratios, Rates, Percents, and Proportions
  - ❖ The Core Processes of Mathematics
  - ❖ Functions and Equations
  - ❖ Measurement

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### Recommendation 6

- ❖ Interventions at all grades should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

❖ Level of Evidence: Moderate

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### Suggestions

- ❖ Provide 10 minutes per session of instruction to build quick retrieval of basic facts. Consider the use of technology, flash cards, and other materials to support extensive practice to facilitate automatic retrieval.
- ❖ For student in K-2 grade explicitly teach strategies for efficient counting to improve the retrieval of math facts.
- ❖ Teach students in grades 2-8 how to use their knowledge of math properties to derive facts in their heads.

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### Recommendation 4

Interventions should include instruction on solving word problems that is based on common underlying structures.

❖ Level of Evidence: **Strong**

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### Suggestions

- ❖ Teach students about the structure of various problem types, how to categorize problems, and how to determine appropriate solutions.
- ❖ Teach students to recognize the common underlying structure between familiar and unfamiliar problems and to transfer known solution methods from familiar to unfamiliar problems.

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**Discussion**

- ❖ How well does your school/district cover critical content? In the core? In intervention materials?
  
- ❖ What steps have you taken or could you take to address the content recommendations in the Practice Guide?

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**Tier 2/3 Instructional Design**

- ❖ Previous Syntheses on mathematics interventions
  - ❖ Not LD specific (Xin & Jitendra, 1999; Kroesbergen & Van Luit, 2003; Baker, Gersten, & Lee, 2002)
  - ❖ New work that is LD specific (Gersten, Chard, Baker, & Jayanthi 2009)

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**Purpose**

- ❖ To synthesize experimental and quasi-experimental research on instructional approaches that enhance the mathematics performance of students with learning disabilities

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### Method

- ❖ Review of all published studies and dissertations between 1970 and 2003
- ❖ Conduct meta-analysis to identify trends in the literature

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### Recommendation 3

Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem-solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

- ❖ Level of Evidence: **Strong**

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### Suggestions

- ❖ Ensure that intervention materials are systematic and explicit and include numerous models of easy and difficult problems with accompanying teacher think-alouds.
- ❖ Provide students with opportunities to solve problems in a group and communicate problem-solving strategies.
- ❖ Ensure that instructional materials include cumulative review in each session.

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### Explicit Instruction

- ❖ Teacher demonstrated a step-by-step plan (strategy) for solving the problem
- ❖ This step-by-step plan had to be problem-specific and not just a generic, heuristic guide for solving problems
- ❖ Students had to use the same procedure/steps shown by the teacher to solve the problem

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### Student Verbalizations

- ❖ Student use of verbalizations while solving problems resulted in large effects
  - ❖ 5 ESs;  $d = 1.25^*$ ; range = 0.23 to 2.49
- ❖ In all studies students verbalized the solution while solving problems
- ❖ In all but 1 study, focus was narrow -- e.g., single digit addition/subtraction; 1-2 step arithmetic story problems involving addition/subtraction
  - ❖ Students did not have to verbalize a range of solutions
  - ❖ In most complex verbalization study -- ES = 0.23

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### Recommendation 5

Intervention materials should include opportunities for the student to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

- ❖ Level of Evidence: **Moderate**

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### Suggestions

- ❖ Use visual representations such as number lines, arrays, and strip diagrams.
- ❖ If necessary consider expeditious use of concrete manipulatives before visual representations. The goal should be to move toward abstract understanding.

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### Use of Visuals

- ❖ Teacher had to use the visual representation during her initial teaching/demonstration OR
- ❖ Students had to use visuals while solving the problem
  - ❖ Could not be an optional step
  - ❖ Visuals were used in solving 1-2 step arithmetic story problems

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### Use of Visuals

- ❖ Student use graphic representations to clarify or solve problems resulted in moderate effect sizes
  - ❖ 5 ESs;  $d = .56^*$ ; range = 0.32 to 1.5
- ❖ Teachers using pictures or diagrams to explain how to solve a problem resulted in moderate effects
  - ❖ 4 ESs;  $d = .55^*$ ; range = -0.38 to 1.15)

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**How do these elements come together? Example ROOTS**

- ❖ 50 lesson program; 20-minute lessons
- ❖ 3 days per week (not consecutive)
- ❖ Delivered in small groups by las
- ❖ Focus on whole number content

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**Example: ROOTS**

Whole Number Content objectives:

- ❖ Use a variety of ways to model and represent numbers (fingers, tallies, ten frame, number line, nifty-fifty chart, base ten blocks)
- ❖ Proficiency in numeration to 20
- ❖ Count, identify, and write numbers to 20
- ❖ Counting from a number other than 1
- ❖ Read and solve simple addition statements
- ❖ Understand place value and model numbers to 20
- ❖ Identify quantities and numbers that are more, less, and equal.

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**Example: Number Rockets**

- ❖ Small groups (11 groups of two students and 16 groups of three students)
- ❖ 3 times per week outside classrooms
  - ❖ Each session: 30 min of tutor-led instruction and 10 min of student practice of math facts
- ❖ Concrete-representational-abstract model
- ❖ Scripted topics addressing focused on whole number understanding

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**Common Elements**

- ❖ Standard protocol approach
  - ❖ Students maintain in the program for a set duration of time
- ❖ Limit scope to critical topics
  - ❖ Whole number
- ❖ Attention to multiple elements of instructional design
  - ❖ Explicit
  - ❖ Visuals/Models
- ❖ Small group instruction

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**Recommendation 8**

- ❖ Include motivational strategies in tier 2 and tier 3 interventions.
  - ❖ Level of Evidence: Low

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**Suggestions**

- ❖ Reinforce or praise students for their effort and for attending to and being engaged in the lesson.
- ❖ Consider rewarding student accomplishment.
- ❖ Allow students to chart their progress and to set goals for improvement.

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**Discussion**

- ❖ How well does your school/district address the instructional design recommendations in the Practice Guide? In the materials you use? In the professional development you provide?
  
- ❖ What steps have you taken or could you take to better address this area?

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**Tier 2/3 Assessment**

- ❖ Use of progress monitoring assessments
  
- ❖ Use of diagnostic assessments

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**Recommendation 7**

- ❖ Monitor the progress of students receiving supplemental instruction and other students who are at risk
  
- ❖ Level of evidence: Low

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### Suggestions

- ❖ Monitor the progress of tier 2, tier 3 and borderline tier 1 students at least once a month using grade appropriate general outcome measures.
- ❖ Use curriculum-embedded assessments in intervention materials to determine whether students are learning from the intervention. Measures can be used as frequently as every day or infrequently as once every week

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### Progress Monitoring Assessments

- ❖ Progress monitoring measures:
  - ❖ Some screening measures have promise as General Outcome Measures but need more research to also be used as progress monitoring measures.
  - ❖ Well researched progress monitoring measures are available for grades one and up.
  - ❖ These possess weaker criterion-related validity than reading measures. (Foegen et al, in press)

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### Diagnostic Assessments

- ❖ Deciding when to use
  - ❖ Prior to intervention or
  - ❖ If the intervention is not successful
- ❖ Sources
  - ❖ Compilation of data from progress monitoring
  - ❖ Curriculum-Based Assessment (e.g. Howell)
  - ❖ In-depth math specific measure (e.g. TEMA)

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### Final thoughts

- ❖ RTI for identification is only possible if tiered support and corresponding elements are in place
- ❖ Professional Development is critical in enhancing both the teaching of mathematics and data-based instructional decision-making
- ❖ Districts and schools should think of developing math specialists similar to reading specialists
- ❖ Our understanding of how best to teach and assess mathematics is rapidly expanding - Stay connected and be flexible in your approach to supporting mathematics achievement

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### Discussion

From where you sit in your current job, was the presentation consistent with how you think about RtI?

Why? Why not?

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### Resources

- ❖ NMAP
- ❖ Center On Instruction – Mathematics
- ❖ National RTI Center
- ❖ Doing What Works
- ❖ Center on Teaching and Learning – University of Oregon

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