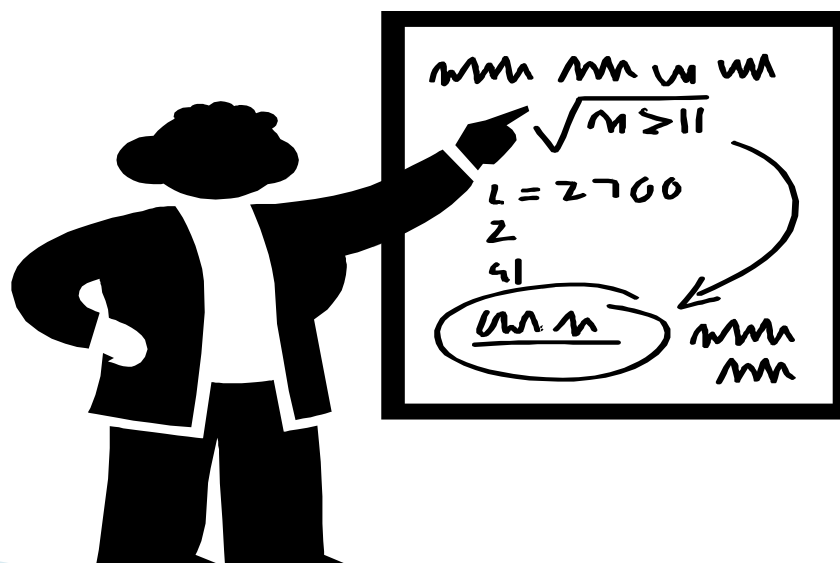


# Math Foundations

A Different Approach to Instruction

Presented by: Dr. Chris Cain



# What the Research Says–



According to the 2009 National Assessment of Educational Progress (NAEP) only 39% of fourth-grade students and only 32% of eighth-grade students scored at the proficient level in mathematics. (NC: 43%;36%)

Feedback?

(National Center for Educational Statistics, 2009)



▶ The National Council for  
Teachers of Mathematics  
(NCTM)

highlights the need for a  
well designed curriculum  
and quality teacher  
preparation.

# Federal Recognition of Lack of Research Studies

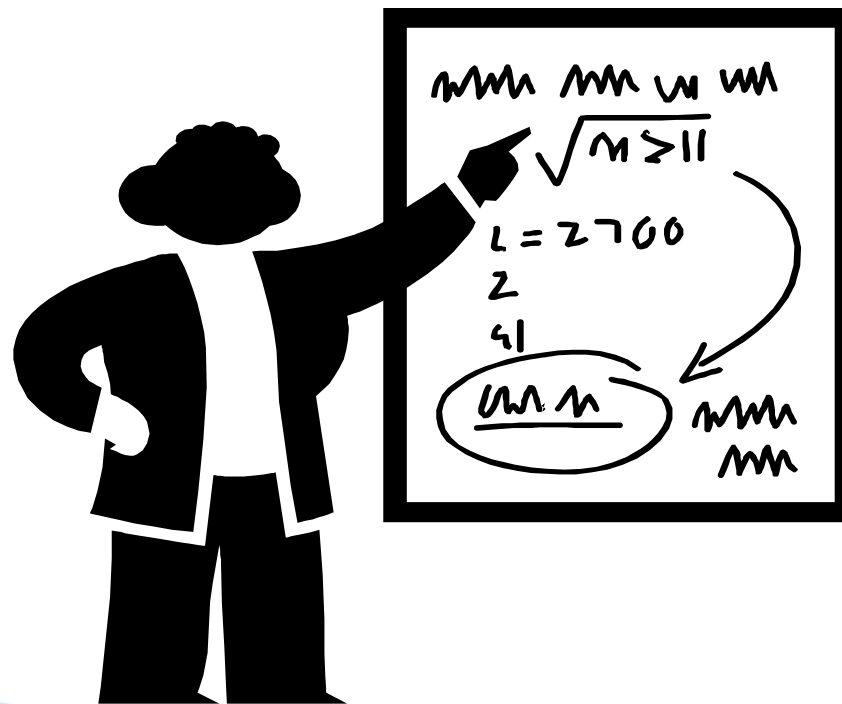
- ▶ IDEIA 2004 and NCLB clearly define a high standard for research-based reading practices
- ▶ IDEIA 2004 did not clearly define a high standard for research-based math practices, because we did not have the same research for math as we did for reading.

What Works clearinghouse:

<http://www.whatworks.ed.gov>

# Math Basics

## International Research



# TIMSS

from Improving Mathematics Instruction (Ed Leadership 2/2004)

## ▶ 1995 Video Study

- Japan, Germany, US
- Teaching Style Implicated

## ▶ 1999 Video Study

- US, Japan, Netherlands, Hong Kong, Australia, Czech Rep.
- Implementation Implicated

# Style vs.. Implementation

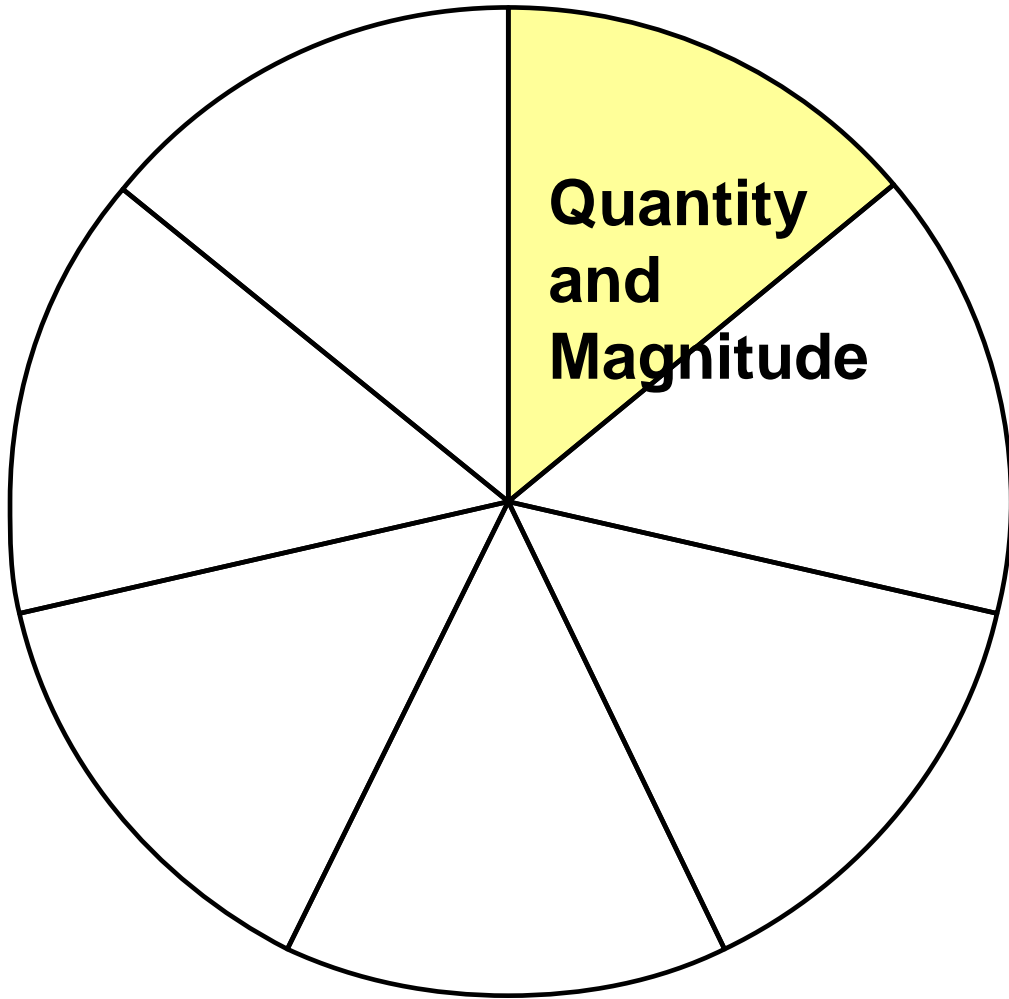
- ▶ High Achieving countries use a variety of styles to teach (calculator vs.. no calculator, 'real-life' problems vs.. 'naked' problems)
- ▶ High Achieving countries all implement connections problems as connections problems
- ▶ U.S. implements connection problems as a set of procedures





## How do we fix the problems?

- ▶ **What are the Components of Number Sense?**
- ▶ **Language Connections!**
- ▶ **Classroom Implementation of the Components of Number Sense.**



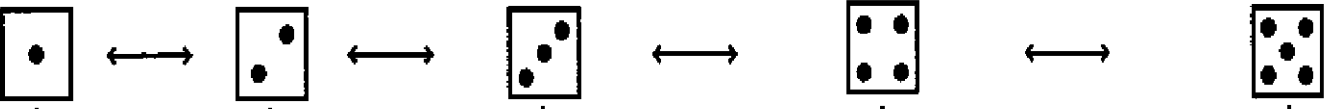
# Different Forms of a Number-- Linking to Magnitude to Number Lines

Representation for Numbers in Different Lands

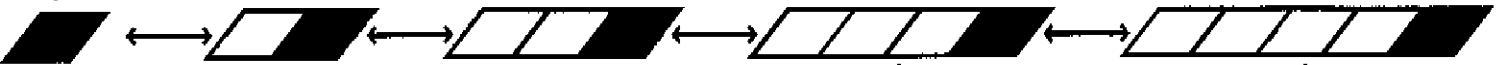
Object Land



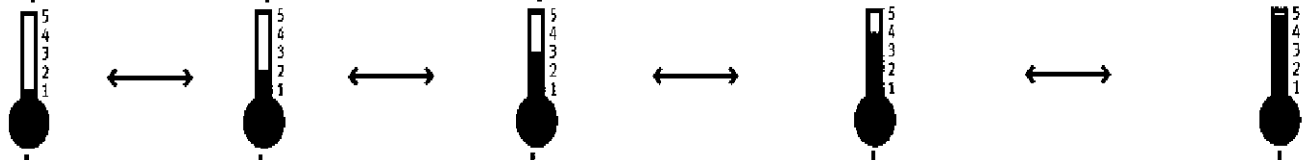
Picture Land



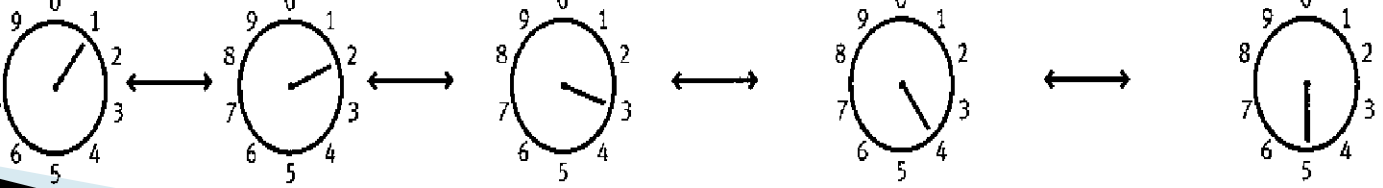
Line Land



Sky Land

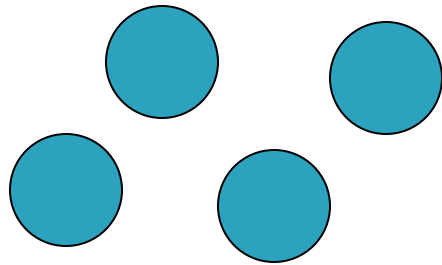


Circle Land



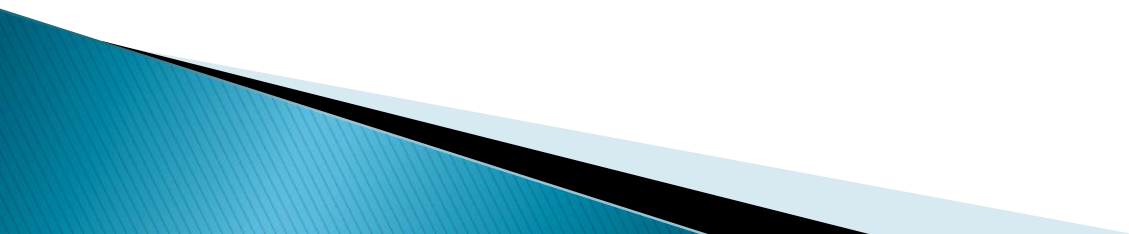
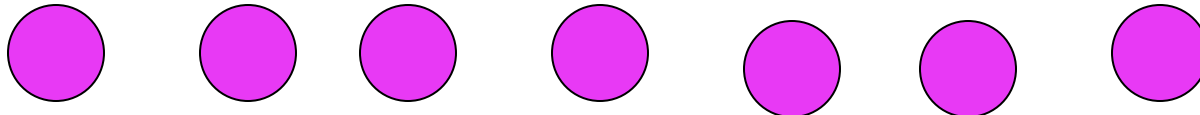
# Doug Clements, Julie Sarama

## Subitizing

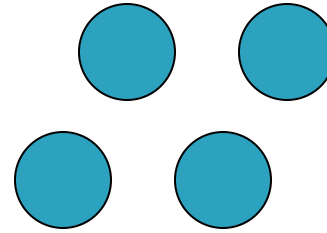
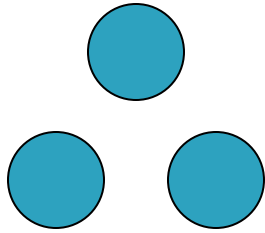


This is a critical skill and may lay underneath early math number sense difficulties with addition and subtraction.

# Number Sense and Instructional Choices

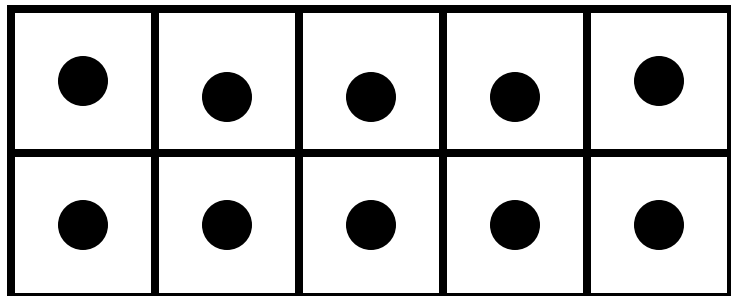


# Number Sense and Instructional Choices



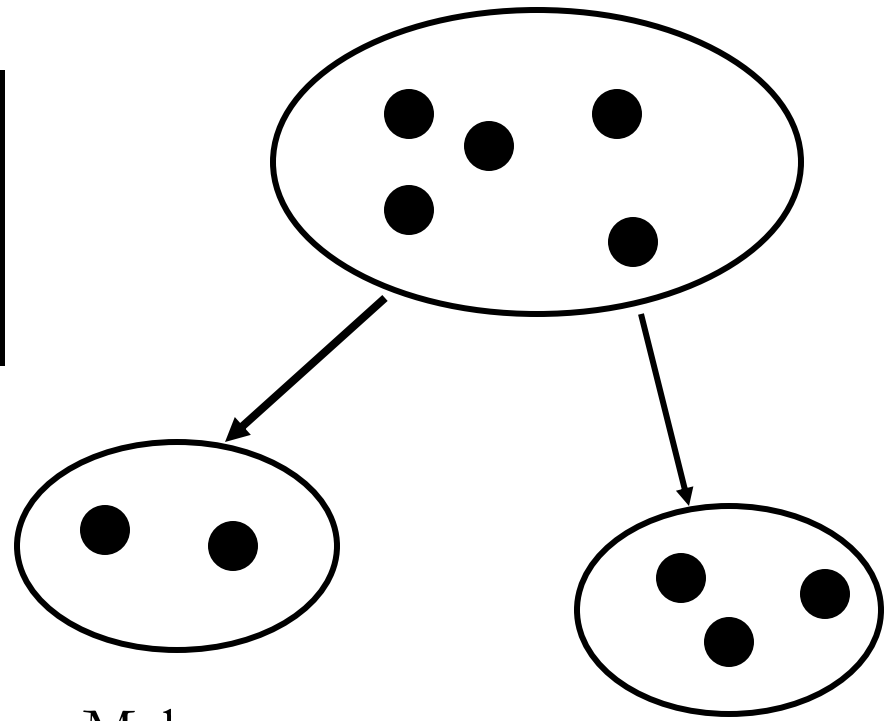
# Making 10: Facts within 20

$$8 + 5$$



$$8 + (2 + 3)$$

$$(8 + 2) + 3$$



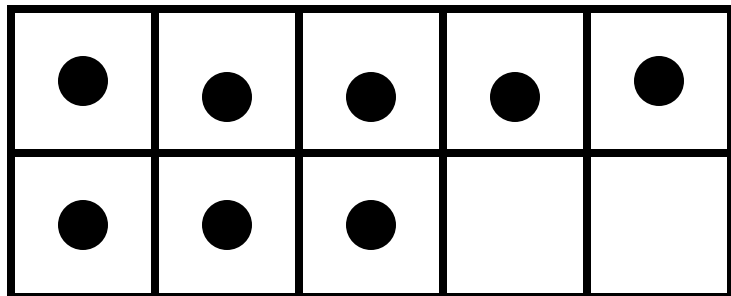
Makes  
Ten

Left Over

1 ten and 3 ones (13)

# Making 10: Facts within 20

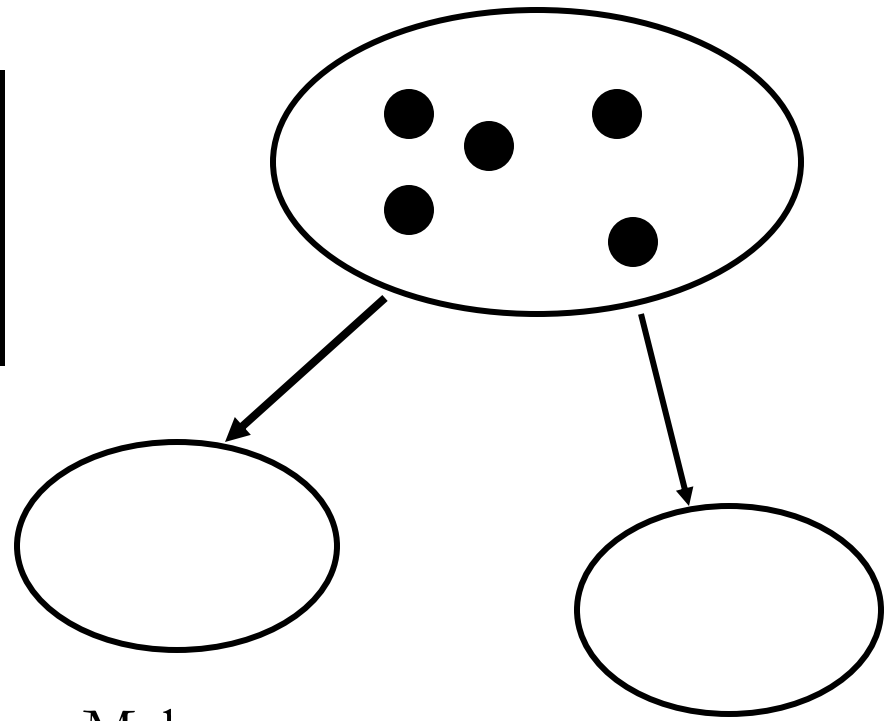
$$8 + 5$$



$$8 + (2 + 3)$$

$$(8 + 2) + 3$$

1 ten and 3 ones (13)



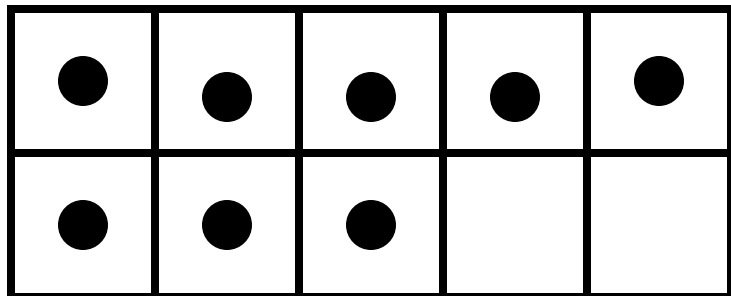
Makes  
Ten

Left Over



# Making 10: Facts within 20

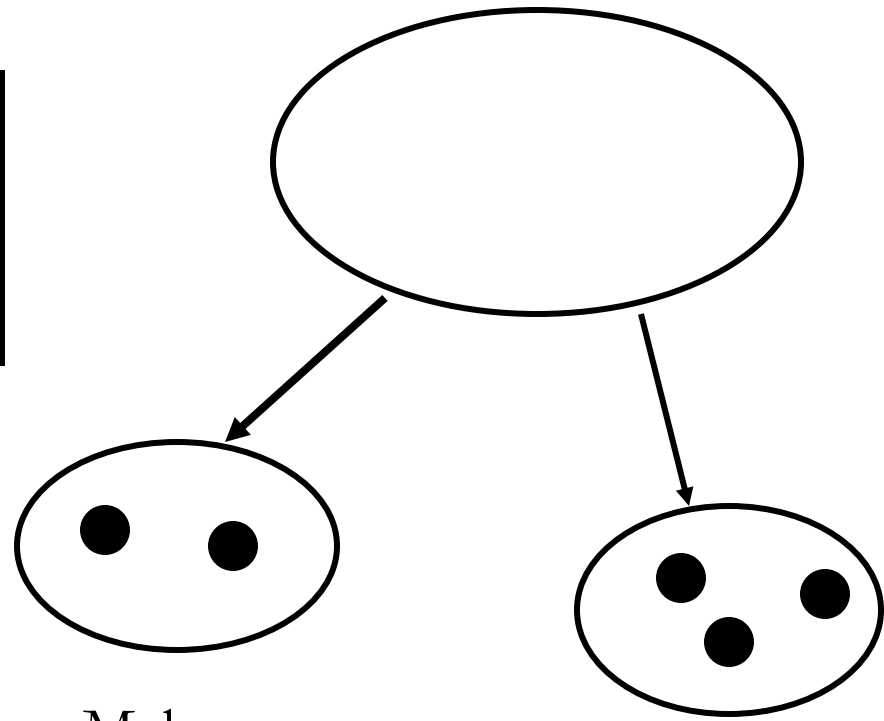
$$8 + 5$$



$$8 + (2 + 3)$$

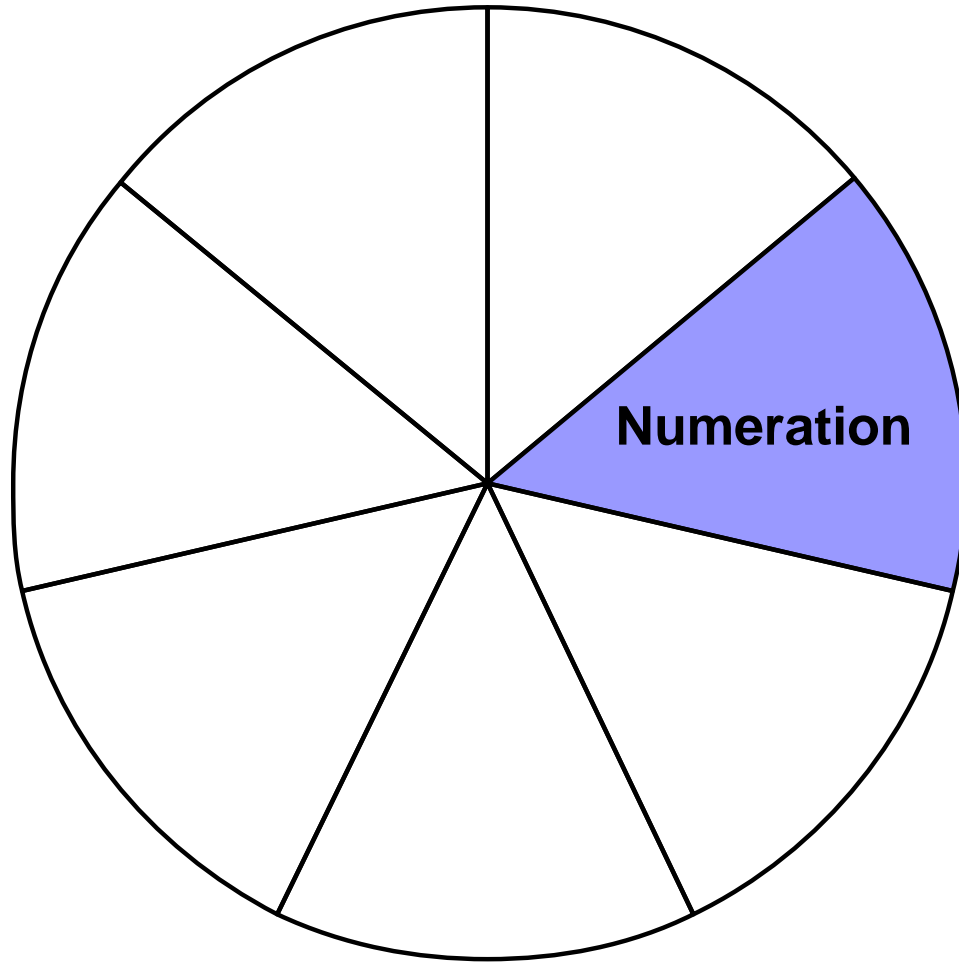
$$(8 + 2) + 3$$

1 ten and 3 ones (13)



Makes  
Ten

Left Over



# Concrete Reality

$$8 - 5 = 8$$

$$7 - 4 = 7$$

# Gelman and Gallistel's (1978) Counting Principles

- ▶ 1–1 Correspondence
- ▶ Stable Order
- ▶ Cardinality
- ▶ Abstraction
- ▶ Order–Irrelevance

# Abstraction

- 3 ones and 2 ones
  - ▶ 3X and 2X
- 3 tens and 2 tens
  - ▶ 3Y and 2Y
- 3 tens and 2 ones
  - ▶ 3X and 2Y
- $\frac{3}{6}$  and  $\frac{2}{6}$
- $\frac{3}{6}$  and  $\frac{2}{5}$

# Think about the power of understanding!

- ▶ Fractions

What is 45% of 80?

What about a child that does not know how to multiply double digit numbers or decimals?

How can we make it clear?

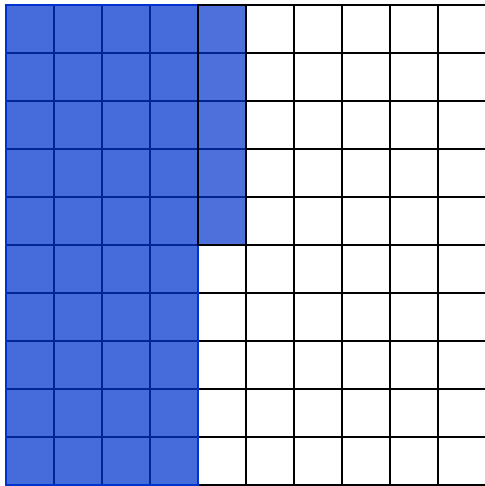
Why are pre-skills so important?

“Move it over two”

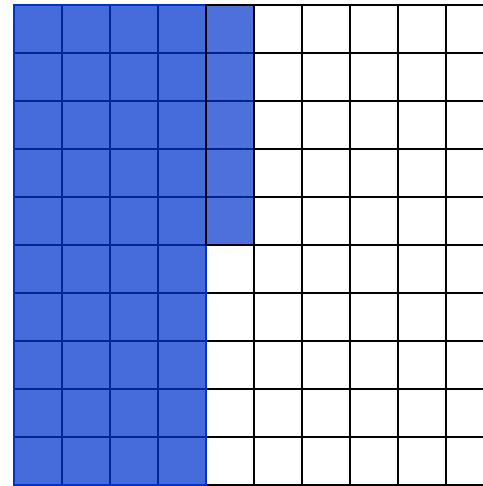
or

percentage and decimal relationship using  
the components of number sense

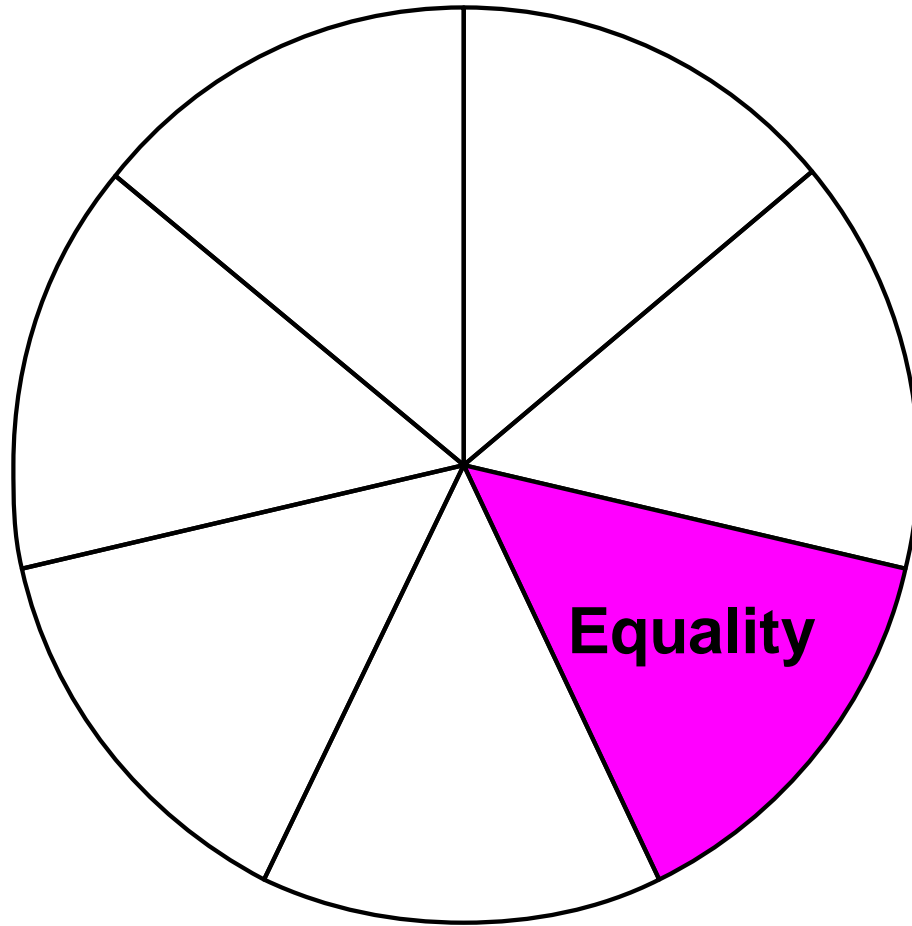
45% and .45



out of 100



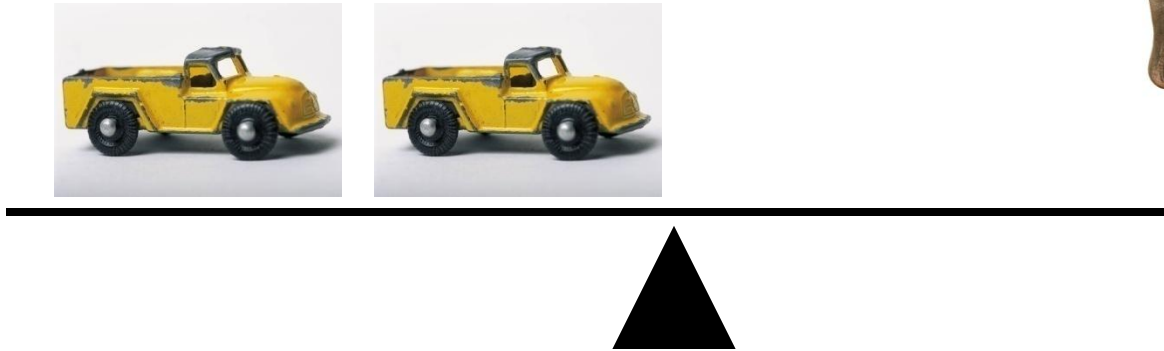
out of 1

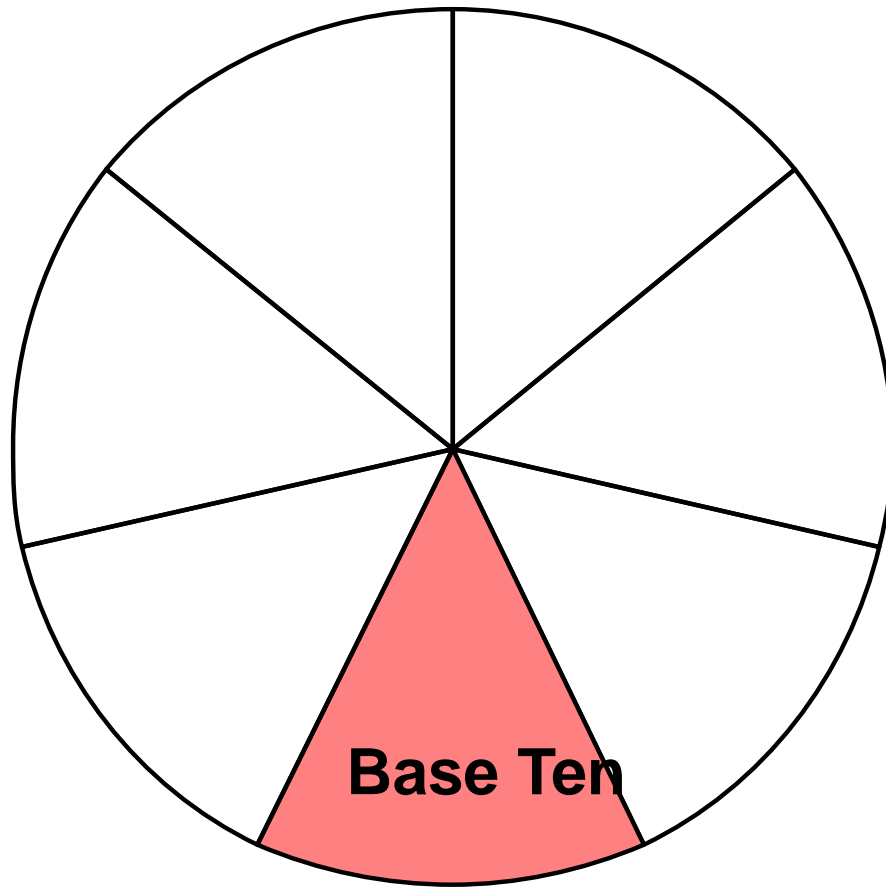




$$1 = 1^3$$

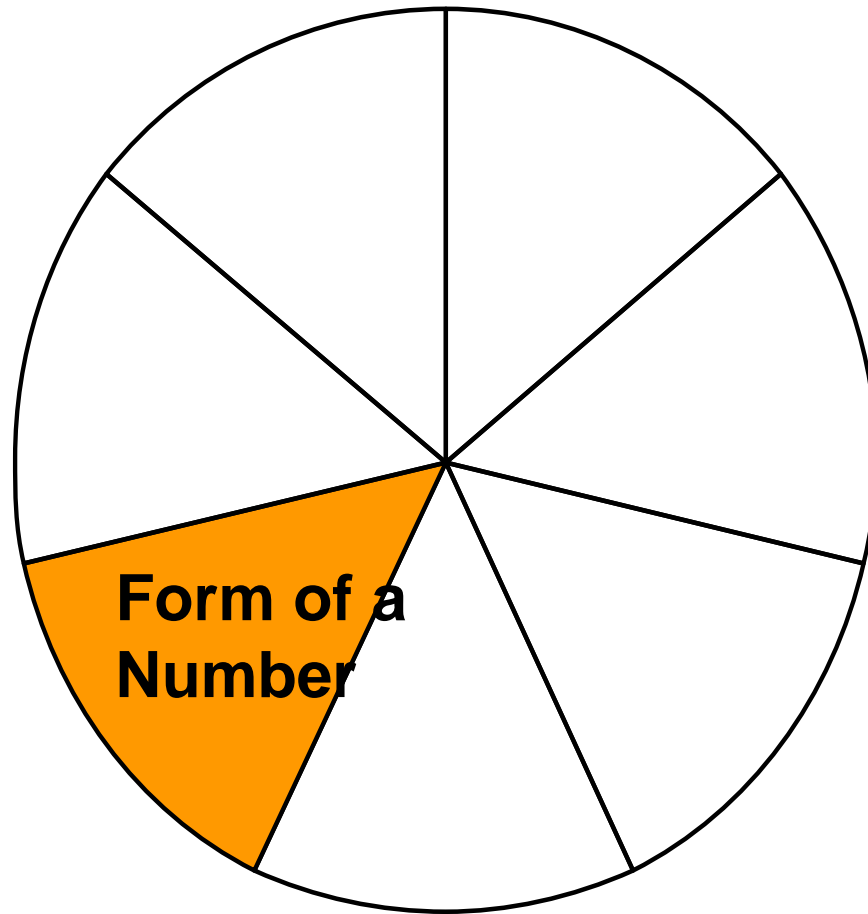
# Are these the same?





	Hundreds	Tens	Units	One tenth	One hundredth	
Weights	$10^2$	$10^1$	$10^0$	$10^{-1}$	$10^{-2}$	
Digits	6	5	4	5	2	
Weighted Value	600	50	4	0.5	0.02	Total 654.52

\* Table taken from Binary Number Systems, 2007  
 Total  
 654.52



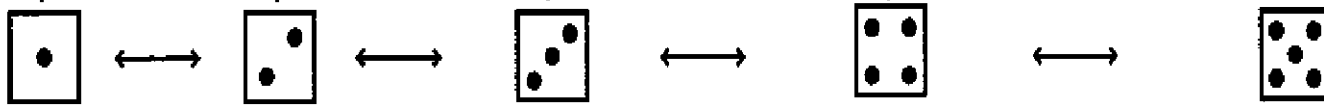
# Different Forms of a Number

## Representation for Numbers in Different Lands

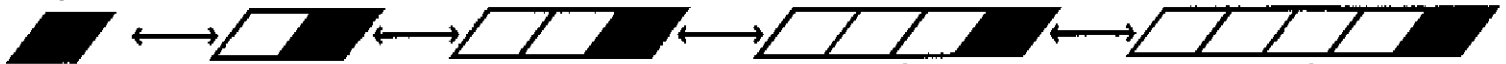
**Object Land**



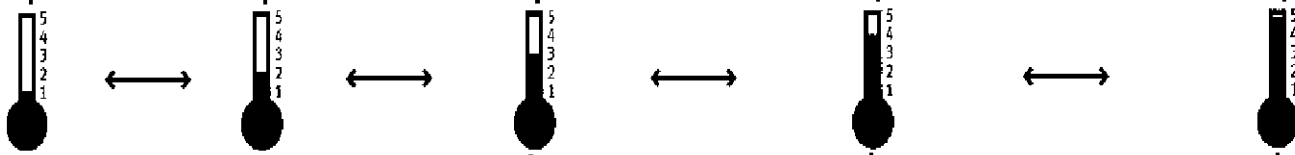
**Picture Land**



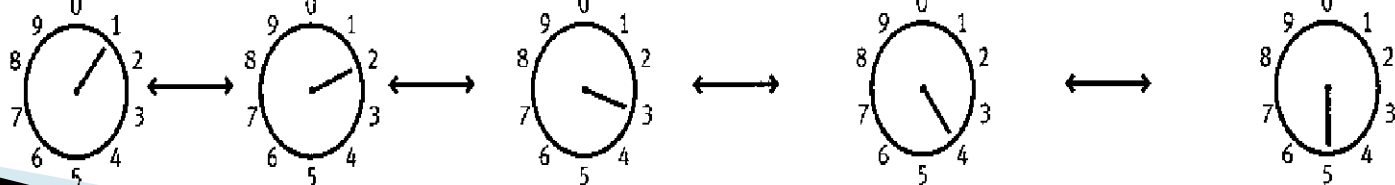
**Line Land**



**Sky Land**

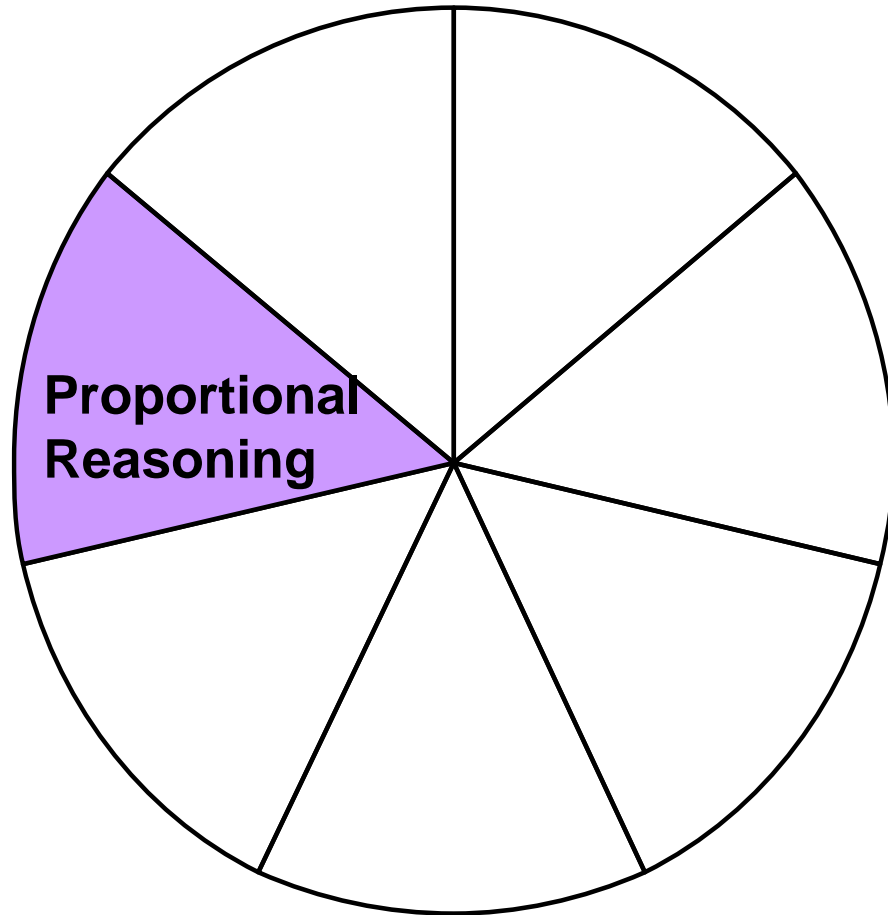


**Circle Land**



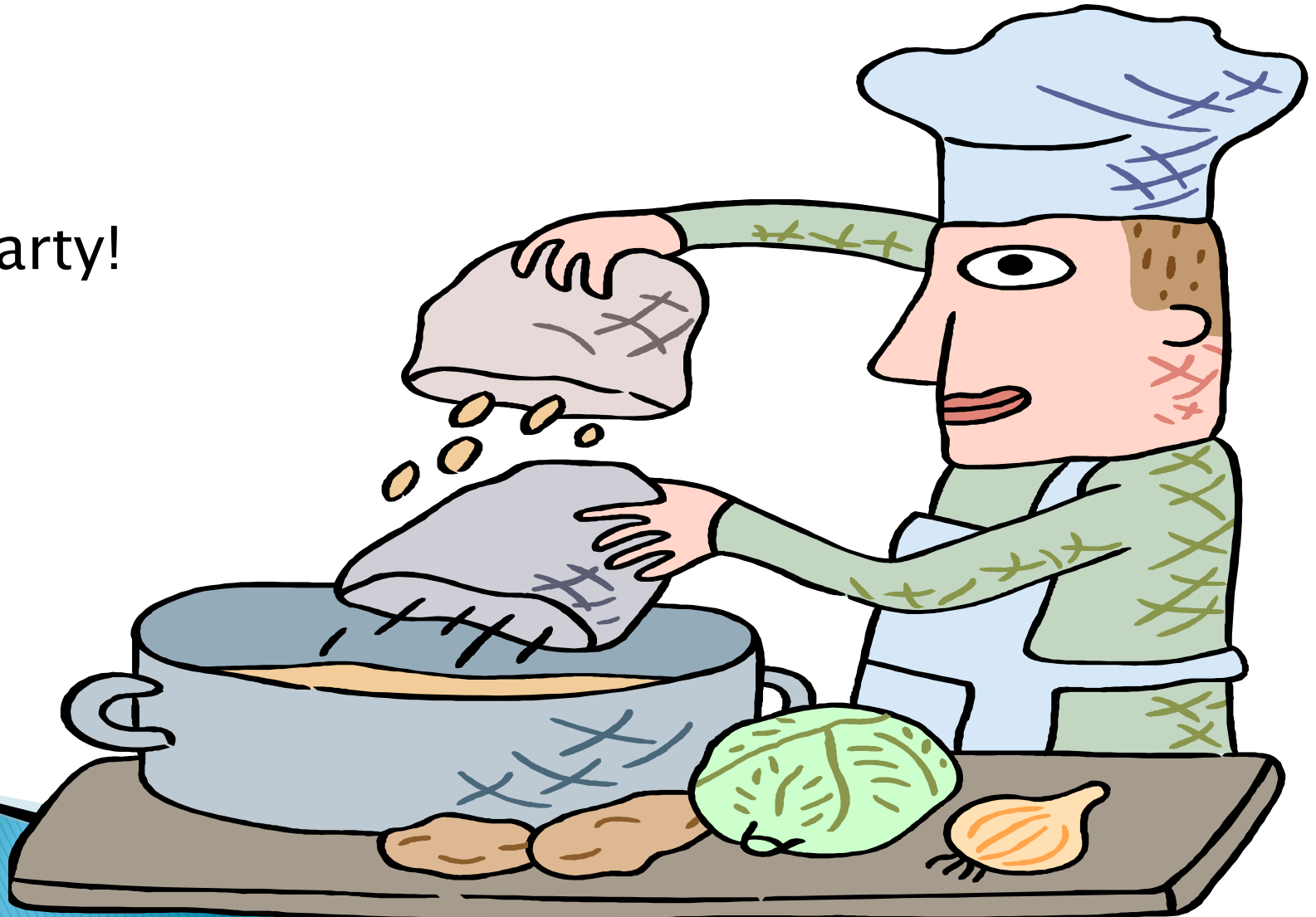
Number Worlds

© 1999

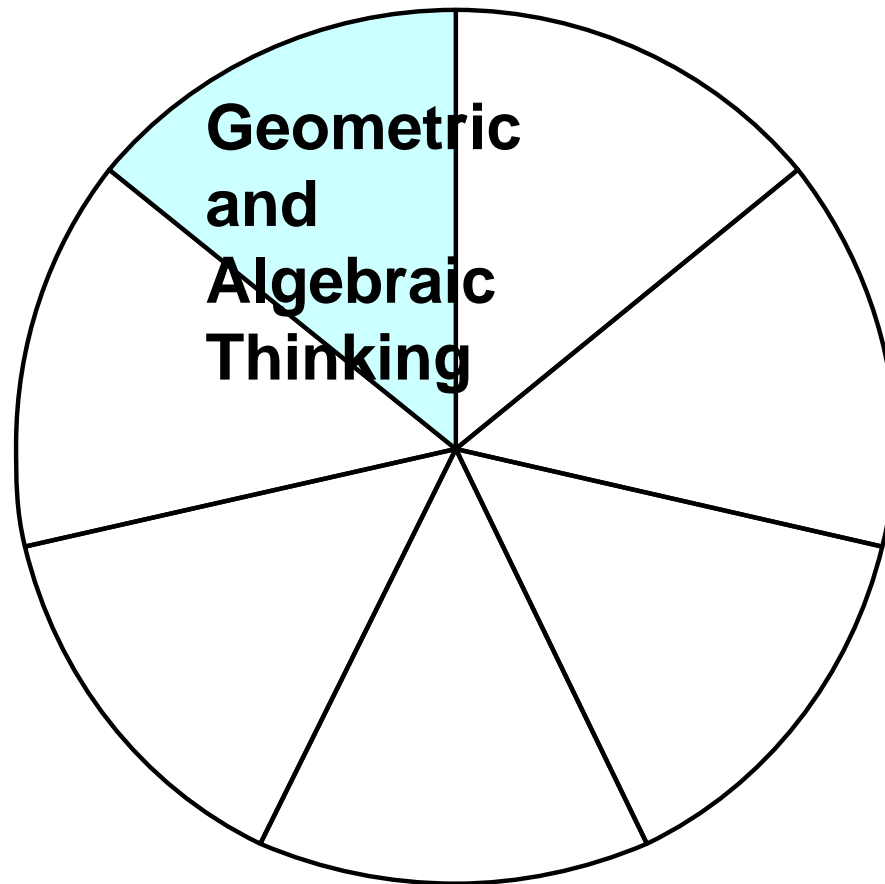


# Proportional Reasoning?

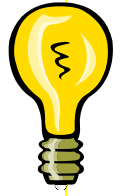
Bean Party!







# Exponents and Geometry



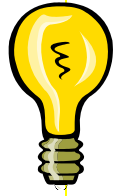
What is  $4^2$ ?

Why is it  $4 \times 4$  when it looks like  $4 \times 2$ ?

It means 'make a square out of your 4 unit side'



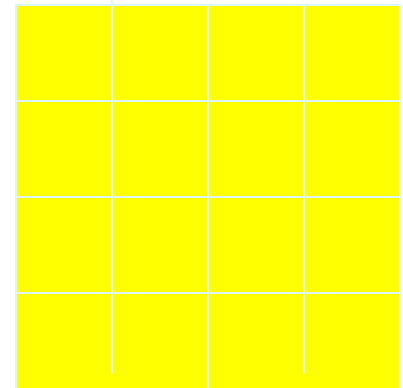
# Exponents and Geometry



What is  $4^2$  ?

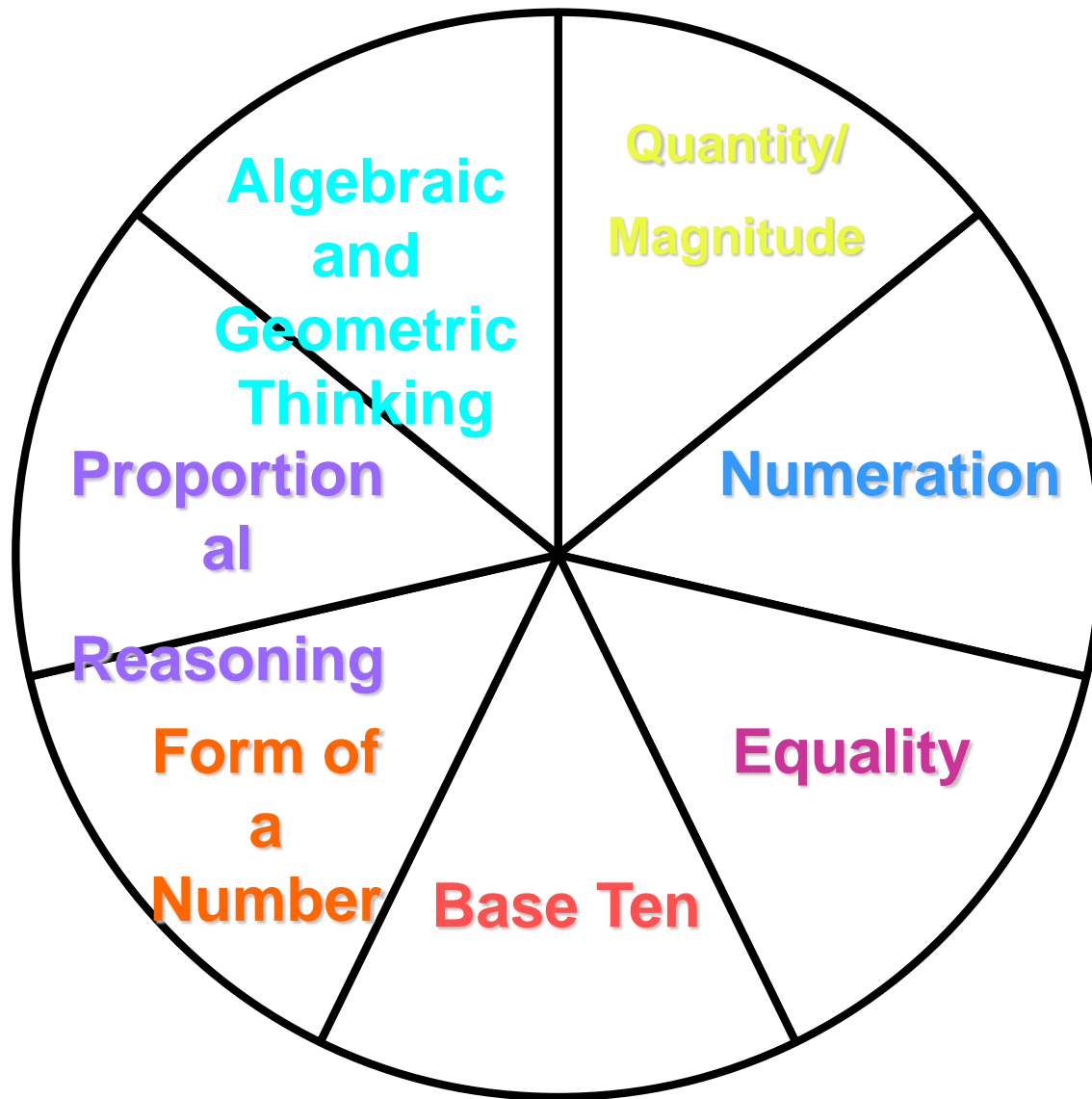
--4 units--

1  
1  
1  
1



You'd get how many little  
1 by 1 inch squares?

$$4^2 = 16$$



**Components of Number  
Sense (almost!)**

# Story One

- ▶ You have a dime ( $1/10$  of a dollar) and seven nickels ( $7/20$  of a dollar).
- ▶ In fractional form show how much money you have all together.

# Story Two

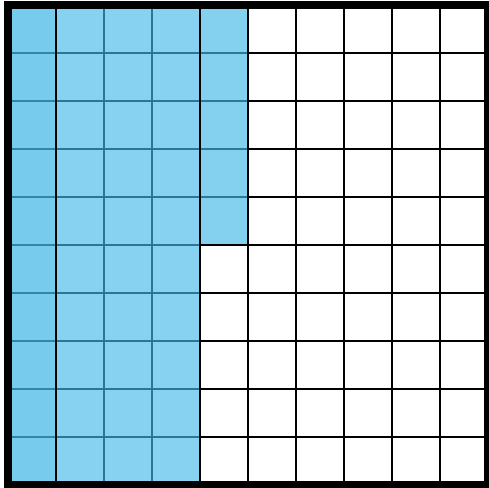
- ▶ A team has two team members
  - 3-point shots  $1/10$  and  $7/20$ .
- ▶ How did the two of them perform as a team ?

Just Do It!



# How Do We Represent the Money Problem Story One?

How do we determine this ratio?

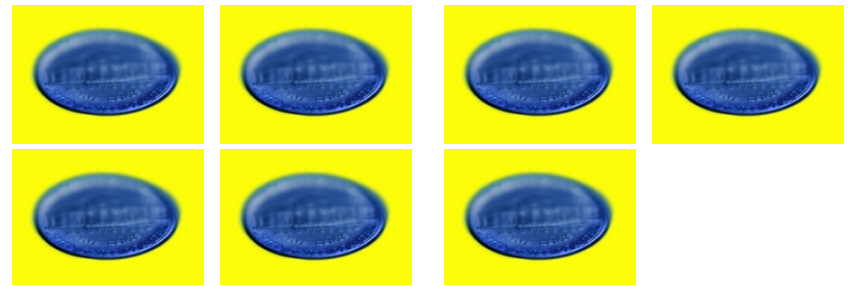


$$\frac{45}{100}$$

1 Dime



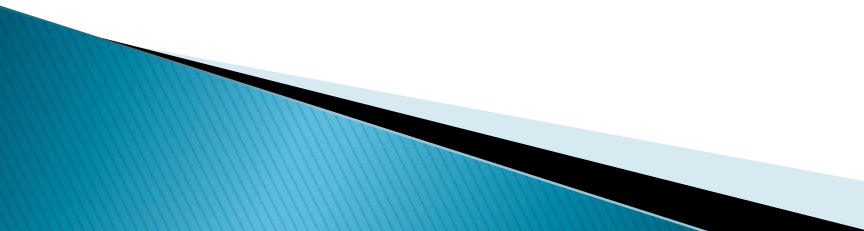
7 Nickels





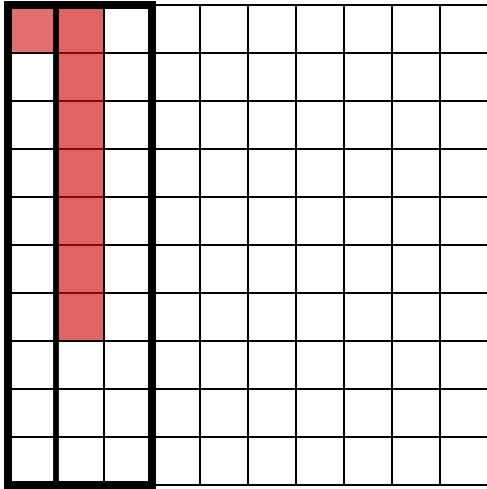
# Abstraction and Unit Size

The unit  
size  
remains  
constant  
with money

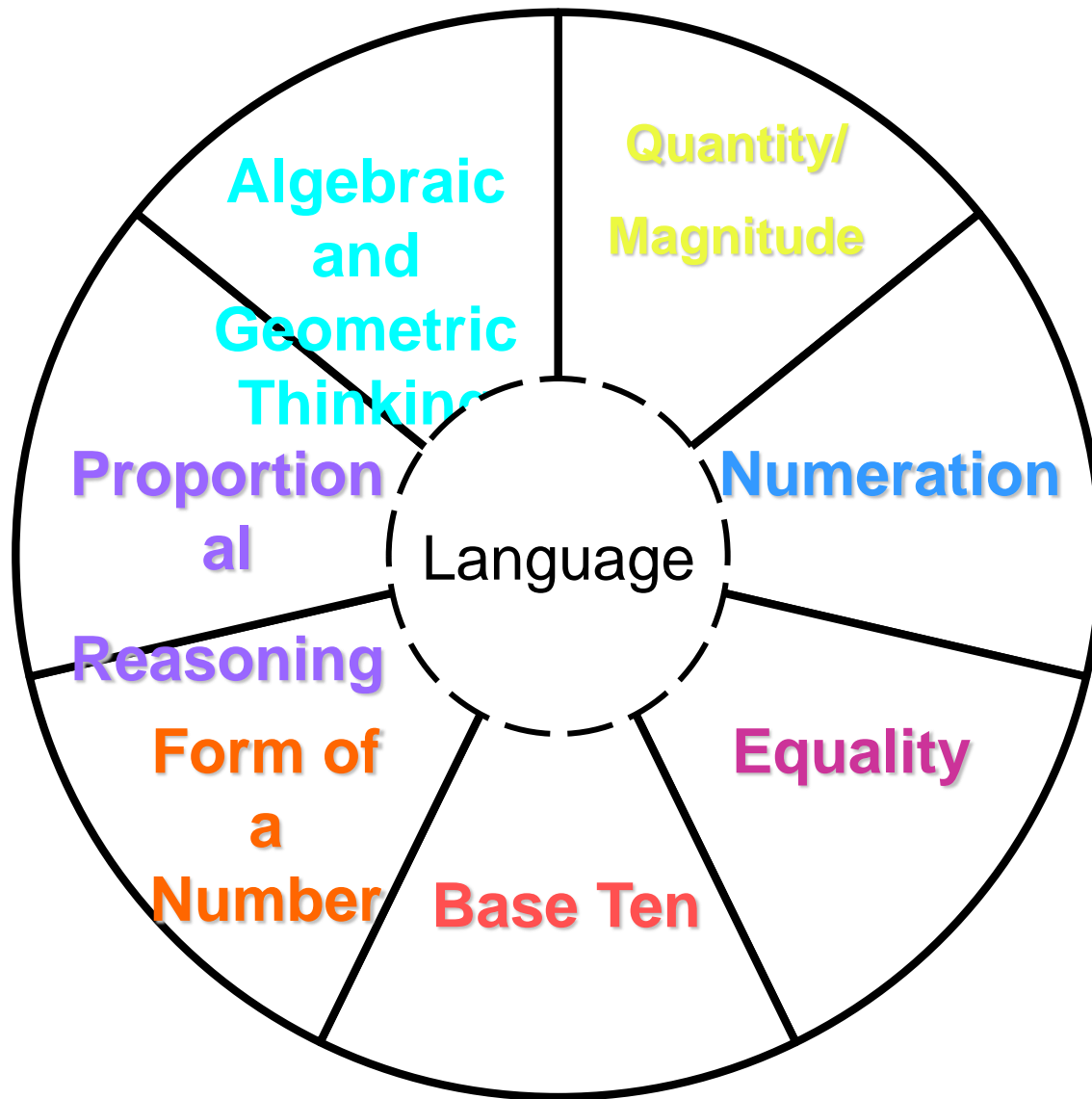


# How Did the Team Do Altogether Story Two?

How do we determine this ratio?

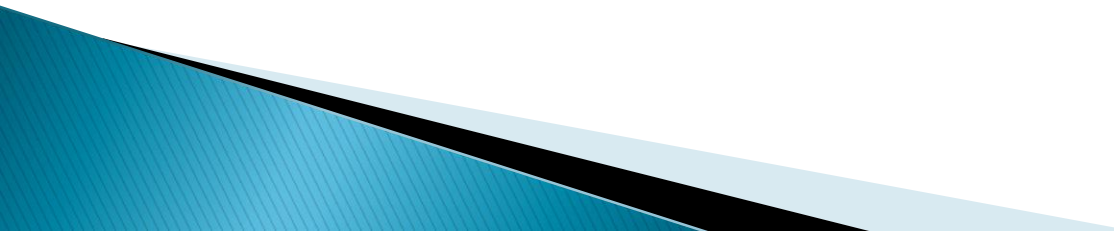


$$\frac{\underline{\quad}8\underline{\quad}}{\underline{\quad}30\underline{\quad}}$$




## Components of Number Sense

# We have to teach math like we teach reading!

- ▶ How is math exactly like a mystery novel?
  - ▶ Do we really try to see the BIG picture?
  - ▶ How does our chapter in the novel fit the whole story?
- 


# Defining Issue in Implementation

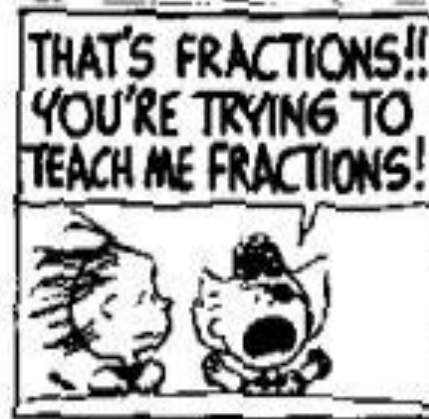
 **Teacher's own  
understanding** of  
mathematics.

Liping Ma

# Knowing and Teaching Elementary Mathematics

Liping Ma

- ▶ Compare and Contrast the pedagogy of Chinese and American Teachers
  - ▶ American Teachers much weaker in Content Knowledge
  - ▶ American teachers teach Procedurally rather than being driven by the logic of the mathematics (implementation)
  - ▶ Ma presents information through teacher responses to elementary math questions
- 



# Problem #3 Division of Fractions

$1 \frac{3}{4}$  divided by  $\frac{1}{2}$

Give a Story Problem to show what is happening with this expression.



# Division of Fractions

U.S. teacher's approach

Flip and multiply

Answers don't match

Confuse multiplying by 2 with  
division by  $\frac{1}{2}$



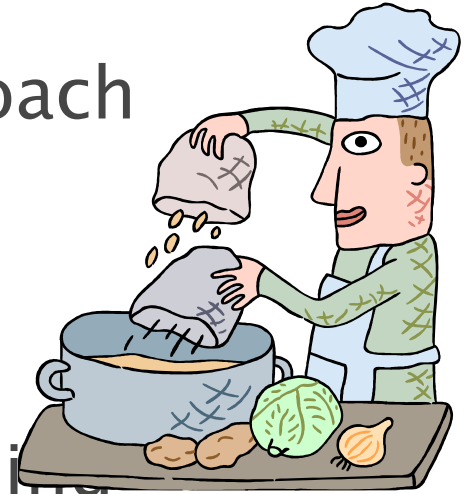
# Division of Fractions

Chinese teacher's approach

Gave mathematically accurate story problem

Explained the mathematics behind the operation

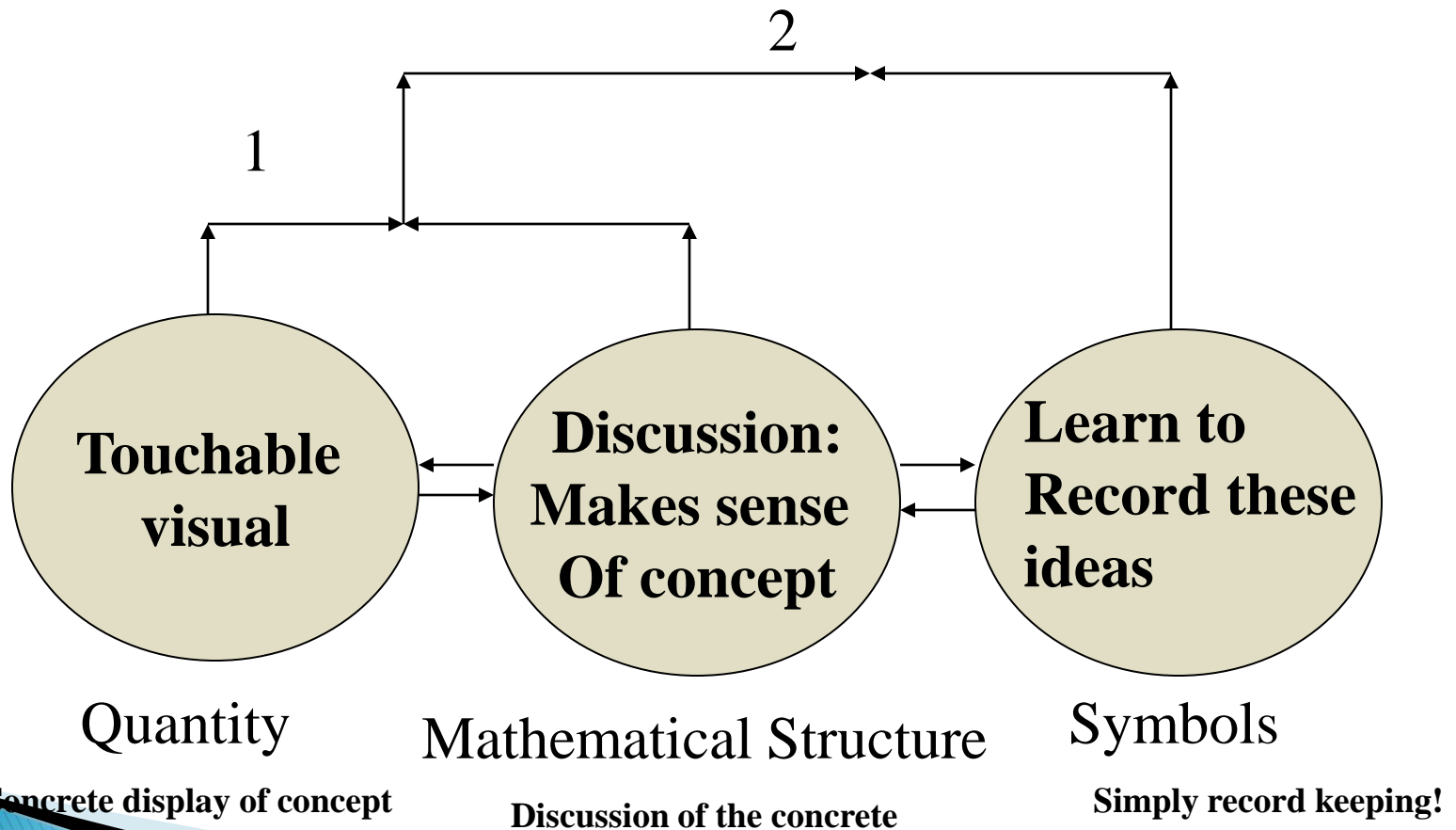
Gave multiple mathematical constructs for division of fractions



# Division of Fractions

- ▶ What does it mean to divide by a fraction?

# Prototype for lesson construction



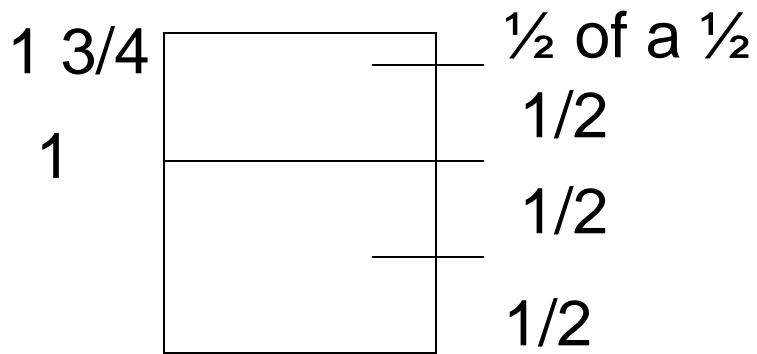
# Division as Repeated Subtraction

Bean Party!

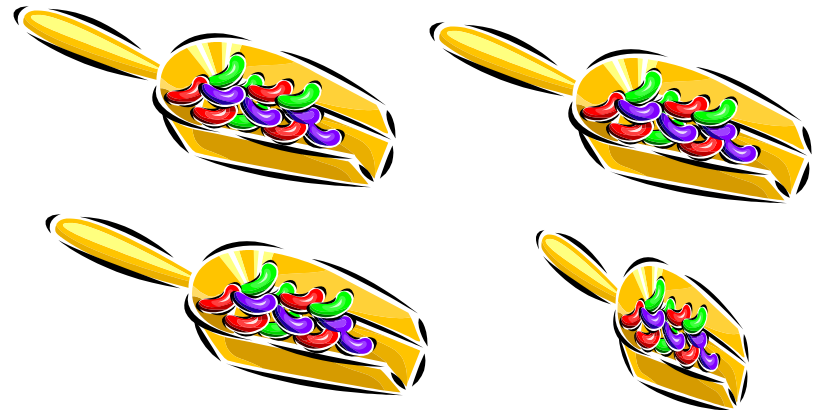


# Division of Fractions: Measurement Model

How many  $\frac{1}{2}$  cups would go into a  $1\frac{3}{4}$  cup container?

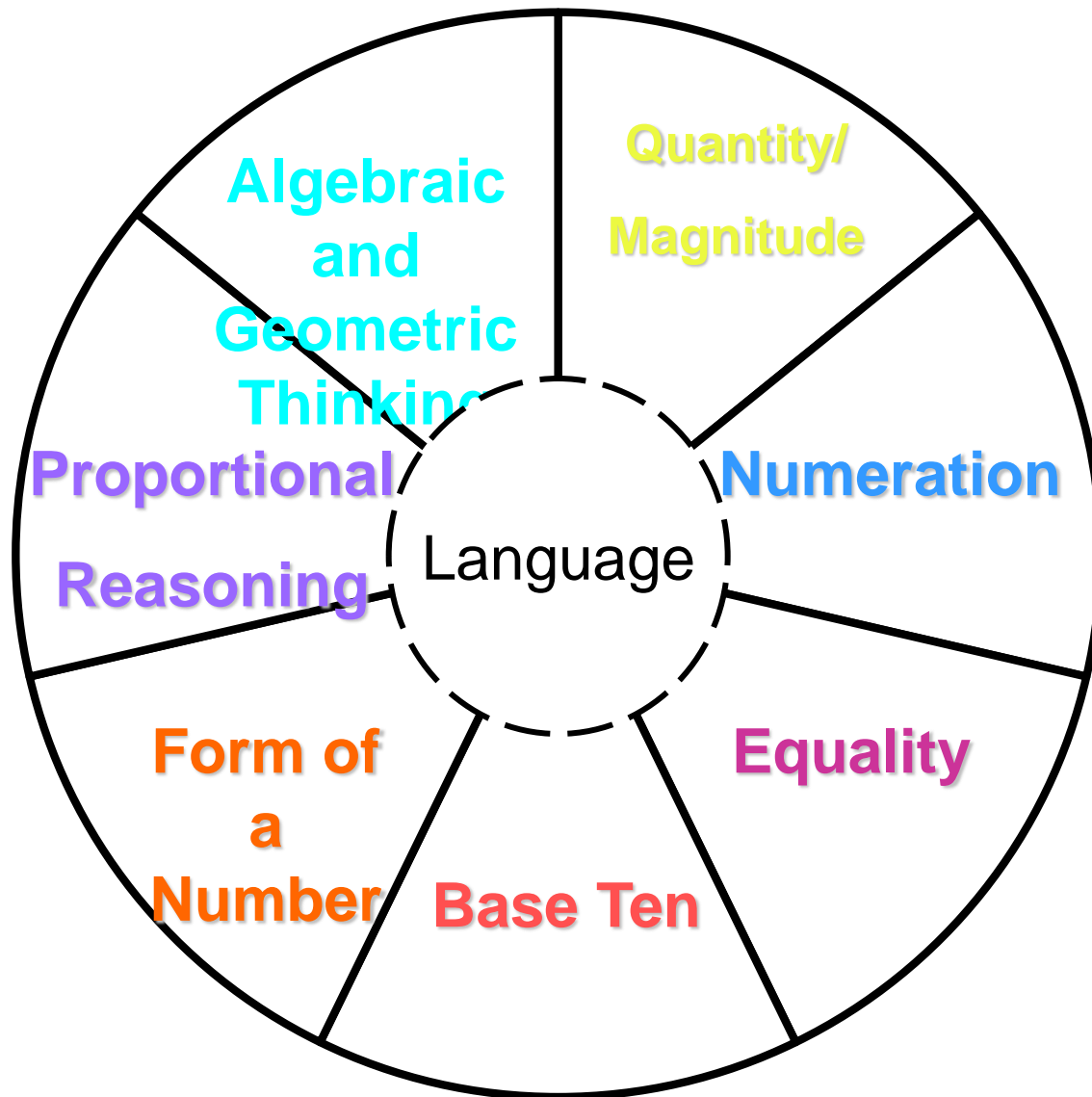


How many  $\frac{1}{2}$  cup scoops?  
How many  $\frac{1}{2}$  cup servings?



Just Do It!

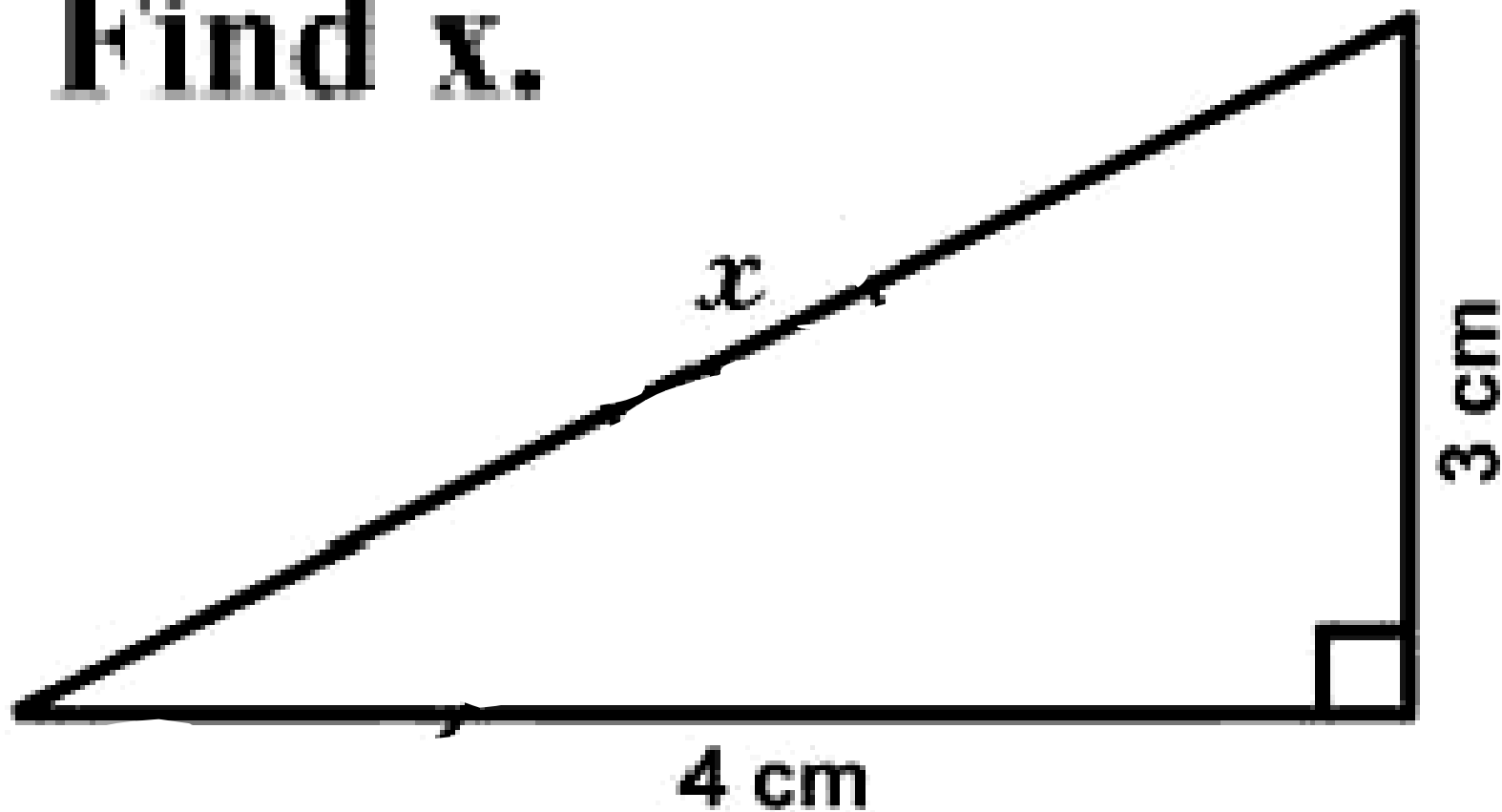




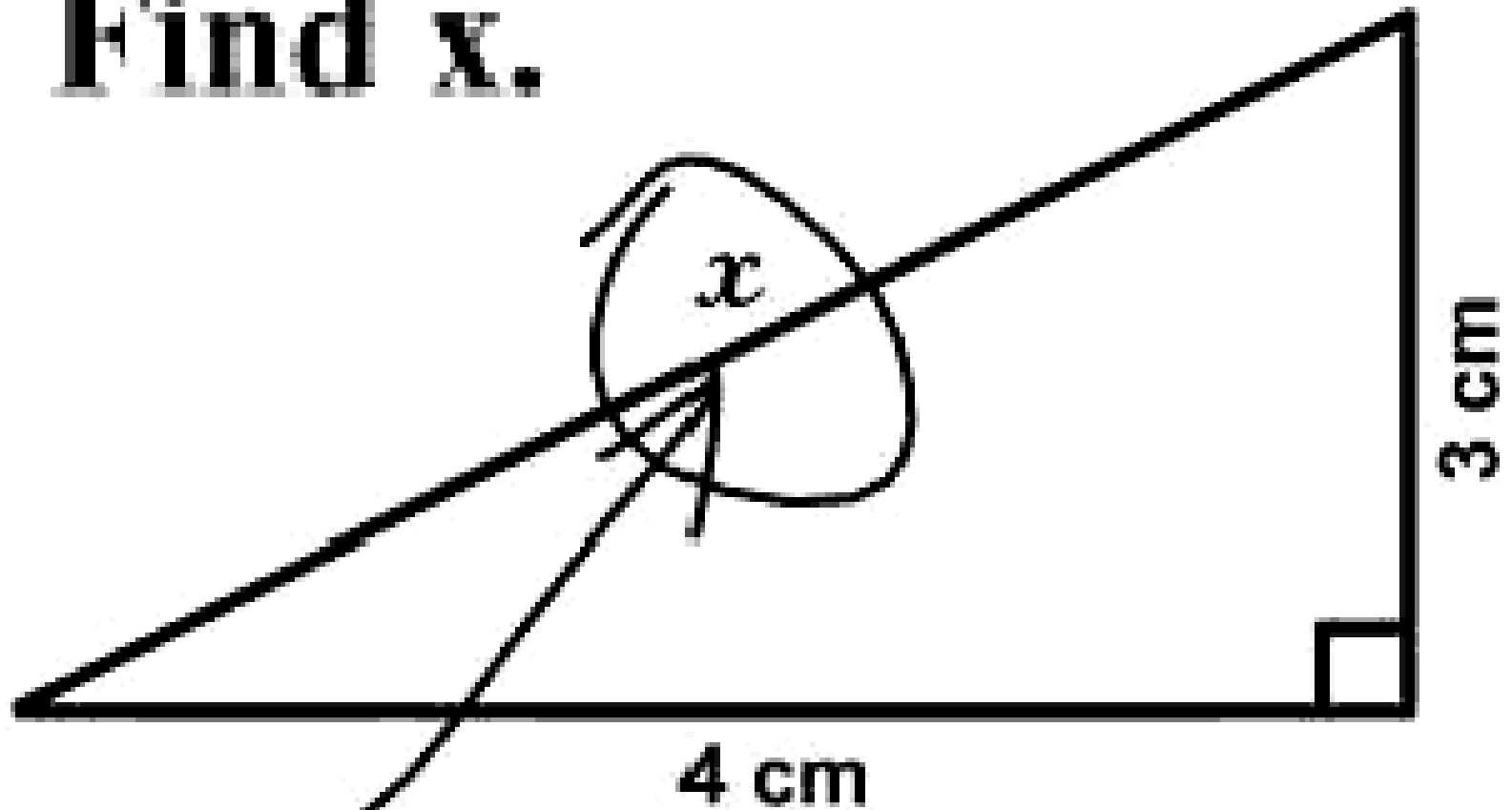
## Components of Number Sense



3. Find  $x$ .

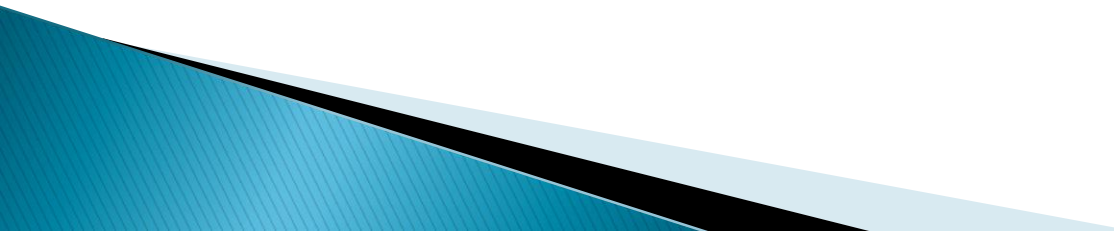


3. Find  $x$ .



*Here it is*

# We can't teach math the way we were taught!

- ▶ Discussion in the classroom
  - ▶ Multiple examples
  - ▶ Make sure we master the new material before we ask children to discriminate (think of a surgeon learning a new procedure).
  - ▶ Homework is made for practicing what we already know.
- 

# Precise and Consistent Language



Sponges!

# Some words about “Key Words”

They don't work...

We tell them—**more** means add

Erin has 46 comic books. She has 18 **more** comic books than Jason has. How many comic books does Jason have.

But is our answer really 64 which is  $46 + 18$ ?

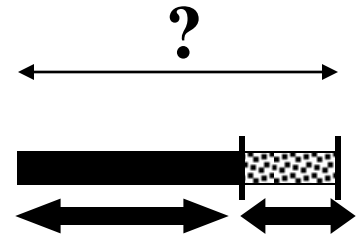


# Structures of Addition

## Join and Part-Part Whole

- There is something, and you get more of it?
- There are two kinds, how many all together?

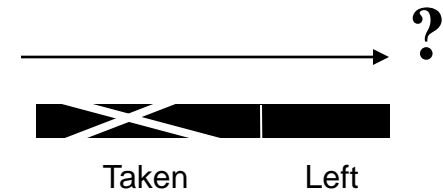
How many altogether?



## Start Unknown

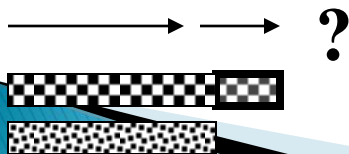
- Some are given away, some are left, how many were there to start?

What did I start with?



## Compare--total unknown

- I know one amount and I have some amount more than that--how many do I have?




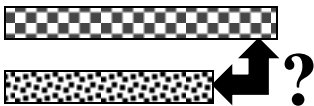
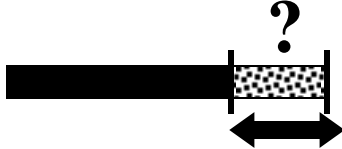
How many do I have?

# Structure

## 3 types of subtraction

Ask yourself if a problem is a subtraction problem—

Does it fit one of these three types?:

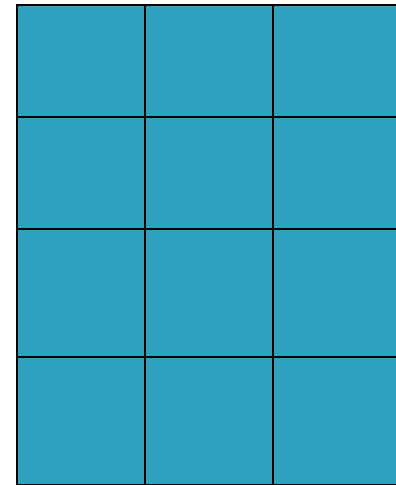
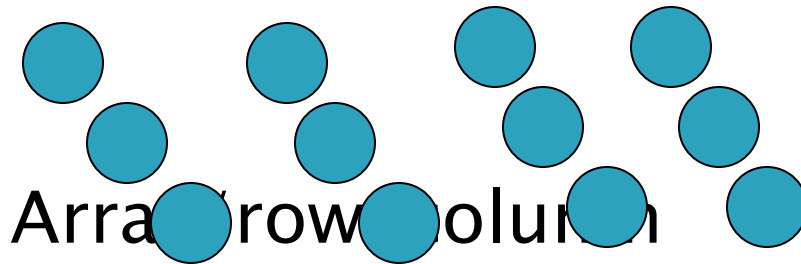
- **The Classic “Take away”**  *You’ve got some amount and “take away” from it: What’s left?*
- **Comparison**  *You compare to see: Who has more or less?*
- **Deficit/Missing amount**  *You need some more to get where you want to be: What is the missing*



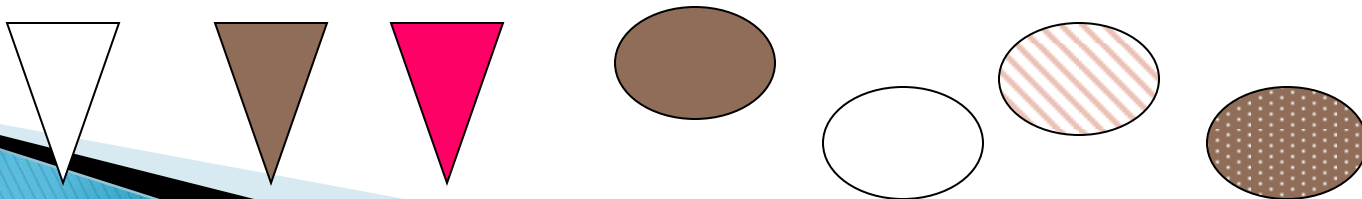
# STRUCTURE:

## 3 Types of Multiplication: 4 x 3

### Repeated Addition



### Counting Principle



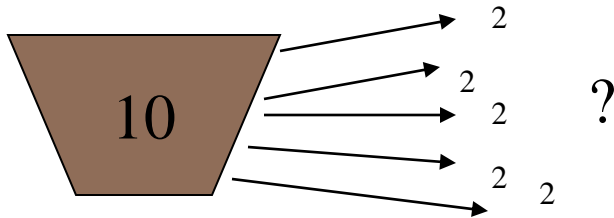
# STRUCTURE:

## 3 Types of Division:

$$10 \div 2$$

### Measurement/Repeated Subtraction

“How many 2s can I get out of 10?”

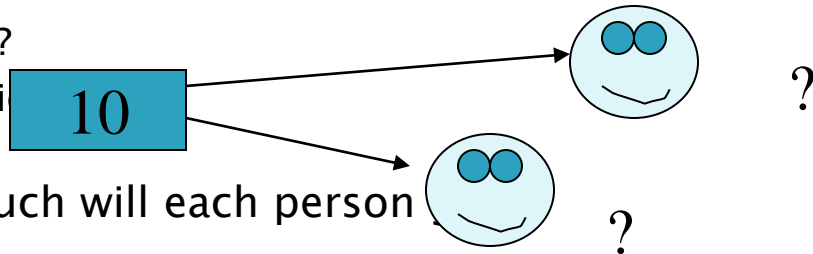


If I have 10 cups of beans and I give out 2 cup portions, how many servings will that provide?

### Partitive/Unitizing/Fair Shares

“How many would 1 person get?”

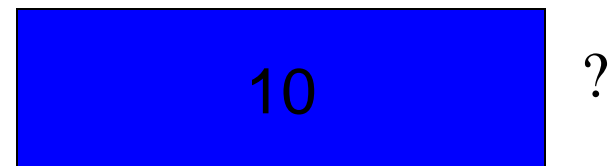
“What would that mean in relation to 10?”



If 2 people find \$10 how much will each person get?

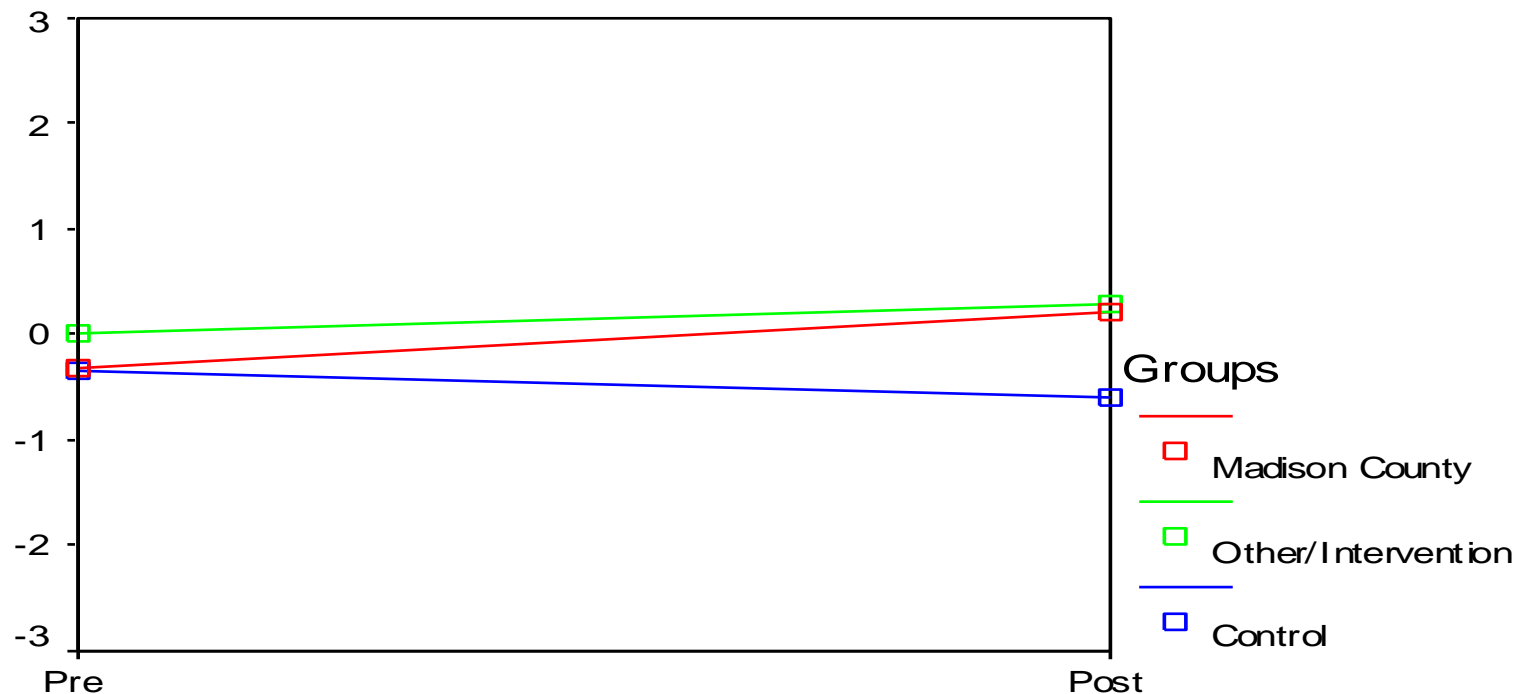
### Product/Factor

“If I have an area of 10 and one side is 2, how long is the other side?”



# Current Mid-Grant Report

Figure 1. Repeated Measures Interaction  
Mathematical Knowledge for Teaching



N = 131 (Madison = 85/ Other = 39/ Control = 7).

# References

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