Early Numeracy for Students with Learning Difficulties

K – 3 content with long term implications

2011 MTSS Symposium

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U.S. Math Performance

The 2005 & 2007 National Assessment of Educational Progress (NAEP) reported:

- 15% of Grade 4 students scored below the basic level
- 25% of Grade 8 students scored below the basic level
- 36% of Grade 12 students scored below the basic level

U.S. Math Performance

The 2005 & 2007 National Assessment of Educational Progress (NAEP) reported:

- 40% of Grade 4 students with disabilities scored below the basic level
- 66% of Grade 8 students with disabilities scored below the basic level
- 83% of Grade 12 students with disabilities scored below the basic level

Mathematics Performance

Translated to Real World Performance

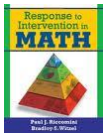
- 78% of adults cannot explain how to compute interest paid on a loan
- 71% cannot calculate miles per gallon
- 58% cannot calculate a 10% tip
- 27% of 8th graders could not correctly shade 1/3 of a rectangle
- 45% could not solve a word problem that required dividing fractions

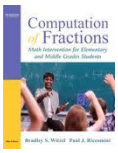
Mathematics Advisory Panel Final Report, 2008

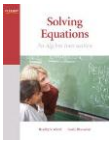
With whom should we first discuss improving math education? Who needs the most help first?

Drawing on Research in Math Education

- IES Practice Guides
- National Math Panel
- Center on Instruction
- National Council of Teachers of Mathematics
- Use these research to set up your non-negotiables for mathematics education






I hope that you leave affected.

What are the non-negotiables in math instruction?

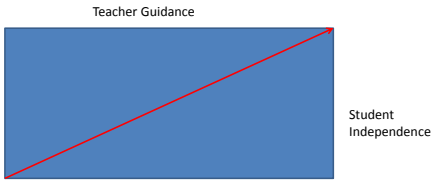
- Time for math
- Types of instruction
  - Modeling
  - Guided Practice
  - Independent Practice / HW
- Engagement and application
- Fluency
- Daily Formative Assessment

Application needs

Instructional Delivery
A focus on math reasoning
Proficiency of basic math skills
  Computation
  Fractions/Decimals
Horizontal Planning
Vertical Planning
Changing Communities

Instructional Delivery

- A continuum of learning is necessary to take students from novice to master.



Number Sense defined

- "a child's fluidity and flexibility in using and manipulating numbers" (Chard et al., 2008, p. 12)
- More precisely, number sense may be "an ability to immediately identify the numerical value associated with small quantities, a facility with basic computing skills, and a proficiency in approximating the magnitudes of small numbers of objects and simple numerical operations" (NMP, 2008, p. 27).

Assessment of Number Sense Components

- Number or Numeral Recognition (Baker et al., 2002; Jordan et al., 2008; Seethaler & Fuchs, 2010)
- Magnitude Comparisons (Chard et al., 2005; Clarke et al., 2008; Seethaler & Fuchs, 2010)
- Counting Principles (Clarke et al., 2008; Lembke & Foegen, 2009; Methe, Hintze, & Floyd, 2008)
- Fact Fluency (Bryant et al., 2008)
- Word Problems (Locuniak & Jordan, 2008)

Intervention Delivery for Number Sense or Early Numeracy

- A pervasive use of concrete objects and number line. Mind Geary's caution (2004).
- While authentic activities are engaging, they often fail at delivering important skill and conceptual understanding (Wu, 1999).
- Thus, how would you like me to show you the interventions?

When things are new, difficult, or dangerous they should be taught
Explicitly
Deliberately
Systematically
(Witzel, 2011)

Numeral and number

Around the tree and around the tree,
that's the way we make a three

Down and over and down some more, that the
that's the way we make a four

Dot notation (Simon & Hanrahan, 2004)

Wisniewski and Smith (2002) found that students with disabilities excelled in speed and accuracy when taught using a dot notation format and transitioned to fluency.

Cardinality – from Numeral and Number

5 = 

3 = 

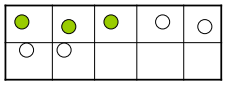
For cardinality - subitization – a fast and accurate observation of number of items

Possibly even with dot notation

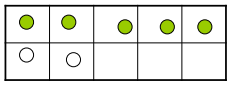
Ten Frames

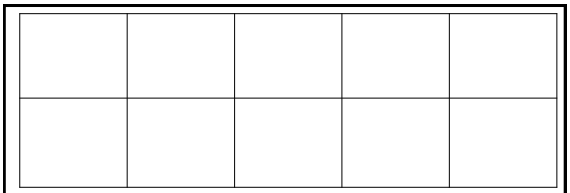
Using patterns to earn numeracy skills and numbers (subitization)

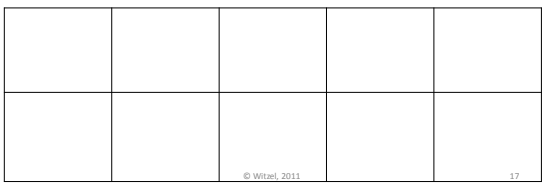
- 3+4=7

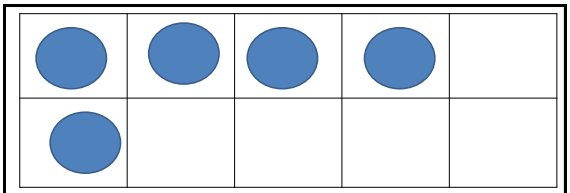


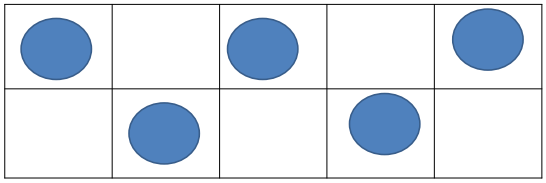
- 5+2=7











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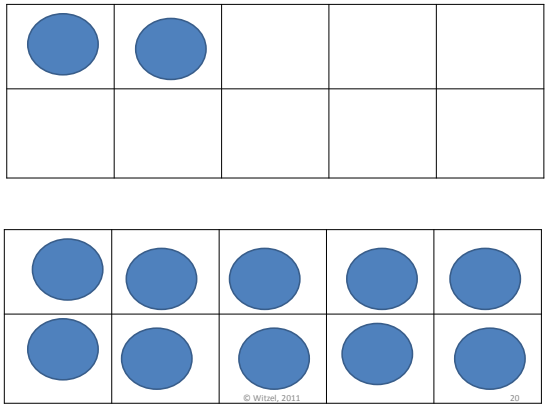

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How could we apply cardinality to counting principles? addition and subtraction facts?

1.

2.

3.

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Place Value is more than expanded notation

- The two most important principles of mathematics are number and place value (Sharma).
- Place value is hard to assess because of its involvement in other math processes and skills
- Common Core
  - K-Working with numbers 11-19 to gain foundations for place value
  - 1-Understand a two-digit number represents amounts of tens and ones
  - 2-Three-digit numbers recognition
  - 3-multi-digit arithmetic

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Examples from the CCSS curriculum document

CC.3.NBT.2 Use place value understanding and properties of operations to perform multi-digit arithmetic. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (A range of algorithms may be used.)

Base 10

0	10	20	***3***
1	11	21	***8***
2	12	22	***13***
3	13	23	***31***
4	14	24	***45***
5	15	25	
6	16	26	
7	17	27	
8	18	28	
9	19	29	

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Base-10 and position recognition

Place Value skills

- 27 = 2 tens and 7 ones
- 45 = 4 tens and 5 ones

- Should be represented physically and verbally
- Advanced learners should use place value within a calculation exercise.

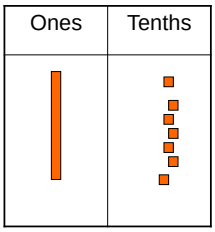
Verbal Place Value Discussions

Counting in place value

1. 2 red boxes and 4 computers
   Summarize what we have.
2. 20 red crayons, 3 erasers, and 5 more red crayons
   Summarize what we have.
3. My first grader at home asked me, "What is 50 plus 1 thousand plus 3 thousand?"
   Four thousand fifty

Language Experience Games

- Race to 100
- Race to 1.00
- See-says



Teach computation with the "why"

Computation and counting principles

- Common Core
  - K-addition and subtraction general definitions
  - 1- add and subtract within 20
  - 2-introduce multiplication
  - 3-introduce division and multiply and divide within 100
- Addition
- Subtraction
- Multiplication
- Division

Common Core

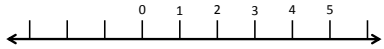
- CC.3.OA.7 Multiply and divide within 100. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. **By the end of Grade 3, know from memory all products of one-digit numbers.**

Students who spend too much energy with basic computation have a difficult time learning the concepts of secondary math.

Some Basic Skills

a) Counting and counting on
- Counting chart, unifix cubes, feed the monkey
- 5+2 or 2+5

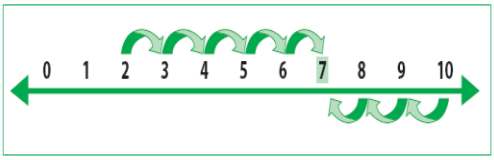
b) Counting backwards (difficult until counting forward)
- Counting chart, unifix cubes
- 5-2



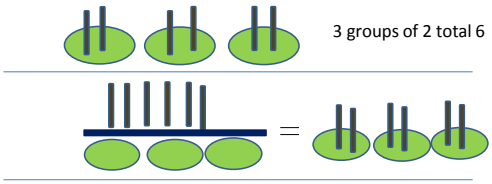
Counting Strategies on a Number line

(Gersten et al, p. 33)

Example 4. Representation of the counting on strategy using a number line



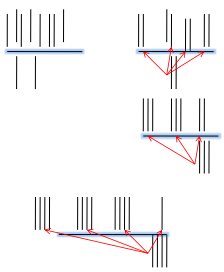
Multiplication and Division



Think of these in terms of physical objects. How would complete 5 ÷ 2

Division to Fractions: concrete

- 8 ÷ 2
- 8 ÷ 3
- 13 ÷ 4



Computational Facility

- Fluency and Automaticity
- Why is it important for students to be able to compute quickly?
- How do we teach speed?
- What is role of calculators in automaticity?

Example Activity

- How were we taught to subtract this problem?

  42
  − 18

- How does this translate to other variations?

  1000
  - 235

- Why?

Benefits of place value knowledge

13 10 + 3
-7 -7
3 + 3 = 6

341 300 + 40 + 1 200 + 130 + 11
-196 -100 – 90 – 6 - 100 – 90 – 6
100 + 40 + 5 = 145

Horizontal Planning: A grade level's success is a team effort

- Key math skills need to be understood by all students.
- Use the CCSS to determine which math skills are benchmarks
- Within the CCSS for math, "the standard algorithm" is used four times. What will be your grade level team's standard algorithm?

Horizontal Planning Activity

- List the steps to multiplication of multi-digit numbers
  Ex. 24 x 76 = or 24
  x76

CCSS 4.NBT.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

- Compare approaches across same grade teacher.
- What does this mean to mathematics instruction at your grade level?

Vertical Planning: One teacher's success depends on the previous teacher's

- The success of each grade level builds upon the next.
- Key math skills need to be understood by all students. To help students grow in math, those key skills can be built across grade levels.
- Within the CCSS for math, "the standard algorithm" is used four times… across three grade levels. Relate each grade's standards to the next. Use work, such as Witzel's (in-press) to set-up progressions.

An example of algorithmic progression

- Second grade "Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends."

4 + 4 + 4 (teaching multiples or skip counting)

			4
			8
			12

- Third grade "Multiplication and division within 100"
- 8 x 6 = ?

6

8	X	X	X	X	X
	X	X	X	X	X
	X	X	X	X	X
	X	X	X	X	X
	X	X	X	X	X
	X	X	X	X	X
	X	X	X	X	X
	X	X	X	X	X

Multiplication Arrays continue

X	50	3
20	1000	60
8	400	24
1000+400+60+24 = 1484		

X	5	.3
2	10	.6
.8	4.0	.24
10+4.0+0.6+0.24		

Conclusion

"A man is like a fraction whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator, the smaller the fraction."

Leo Tolstoy

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